MATHEMATICS

OXFORD MATHS 9

STAGE 5

SAMPLE CHAPTER

UNCORRECTED PAGE PROOFS

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NSW CURRICULUM

2

Indices

Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Prime factorisation
- ✔ Multiplying terms
- ✔ Dividing terms
- ✔ Rounding integers
- ✔ Ordering and comparing large numbers

Curriculum links

- Simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases [MA5-IND-C-01]
  - Extend and apply the index laws to variables, using positive-integer indices and the zero index
  - Simplify algebraic products and quotients using index laws
  - Apply index laws to numerical expressions with negative-integer indices
- Solves measurement problems by using scientific notation to represent numbers and rounding to a given number of significant figures [MA5-MAG-C-01]
  - Identify and describe very small and very large measurements
  - Estimate and round numbers to a specified degree of accuracy
  - Express numbers in scientific notation

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Materials

✔ Calculator

2A Indices

Learning intentions

By the end of this topic you will be able to ...

✔ convert between index notation and expanded form
✔ calculate the value of numbers in index notation
✔ express integers as a product of prime factors.

Inter-year links

Year 7	1G Indices and square roots
Year 8	4A Indices
Year 10	2A Indices

Index notation

- **Index notation** (or **index form**) is used to represent repeated multiplication.
  - → 3^4 is read as '3 to the power of 4'.
  - → a^3 is read as 'a to the power of 3'.
- The **base** is the number or variable that is multiplied repeatedly.
- The **index** (or **exponent**) indicates the number of times the base is multiplied.
- If no index is indicated, then the base has an index of 1.

$$2 = 2^1$$
$$x = x^1$$

index/exponent

base → $\underbrace{3^4}_{\text{index notation}} = \underbrace{3 \times 3 \times 3 \times 3}_{\text{expanded form}} = \underbrace{81}_{\text{basic numeral}}$

index/exponent

base → $\underbrace{a^3}_{\text{index notation}} = \underbrace{a \times a \times a}_{\text{expanded form}}$

- Index notation can also be used to represent powers of negative numbers.

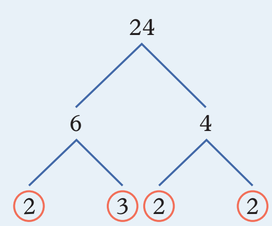
index/exponent

base → $\underbrace{(-2)^3}_{\text{index notation}} = \underbrace{(-2) \times (-2) \times (-2)}_{\text{expanded form}} = \underbrace{-8}_{\text{basic numeral}}$

  - → If the base is negative and the index is an even number, the basic numeral will be positive.
  - → If the base is negative and the index is an odd number, the basic numeral will be negative.

Prime factorisation

- The **prime factorisation** of a positive integer is the product of all prime factors of that integer.
  - → Prime factorisation is often expressed in index notation with the bases listed in ascending order. For example, the prime factorisation of 24 is: $24 = 2 \times 2 \times 2 \times 3$
    $= 2^3 \times 3$.
- The prime factorisation of a positive integer can be found using **factor trees**. In factor trees, composite numbers are broken down into pairs of factors until all remaining factors are prime numbers.



Example 2A.1 Calculating the value of a number in index notation

Write the following in expanded form and evaluate.

a 2^5 **b** $(-4)^3$ **c** $\left(\frac{2}{5}\right)^4$

THINK	WRITE
a 1 Identify the base and the index. The base is 2 and the index is 5, so 2 is multiplied by itself 5 times.	**a** $2^5 = 2 \times 2 \times 2 \times 2 \times 2$
2 Perform the multiplications.	$= 4 \times 2 \times 2 \times 2$ $= 8 \times 2 \times 2$ $= 16 \times 2$ $= 32$
b 1 Identify the base and the index. The base is -4 and the index is 3, so -4 is multiplied by itself 3 times.	**b** $(-4)^3 = -4 \times -4 \times -4$
2 Perform the multiplications. Recall that if the base is negative and the index is an odd number, then the basic numeral will be negative.	$= 16 \times -4$ $= -64$
c 1 Identify the base and the index. The base is $\frac{2}{5}$ and the index is 4, so $\frac{2}{5}$ is multiplied by itself 4 times.	**c** $\left(\frac{2}{5}\right)^4 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
2 Perform the multiplications. Recall that to multiply fractions, you multiply the numerators together and the denominators together.	$= \frac{2 \times 2 \times 2 \times 2}{5 \times 5 \times 5 \times 5}$ $= \frac{16}{625}$

Example 2A.2 Writing variables in expanded form

Write the following in expanded form.

a x^4 **b** $(-ab)^3$ **c** $2xy^2z$

THINK	WRITE
a Identify the base and the index. The base is x and the index is 4, so x is multiplied by itself 4 times.	**a** $x^4 = x \times x \times x \times x$
b Identify the base and the index. The base is $-ab$ and the index is 3, so $-ab$ is multiplied by itself 3 times.	**b** $(-ab)^3 = -ab \times -ab \times -ab$
c There are four bases in this term. Identify the bases and the matching index. Recall that if a base doesn't have an indicated index, then the index is 1. Therefore, 2, x and z each have an index of 1, and y has an index of 2.	**c** $2xy^2z = 2 \times x \times y \times y \times z$

Example 2A.3 Prime factorisation using factor trees

Use a factor tree to express each number as the product of its prime factors. Write your answers in index notation.

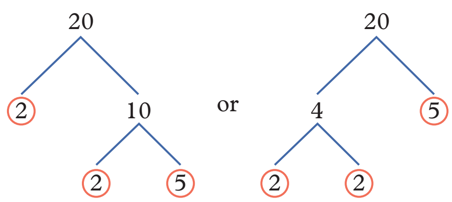
a 20 **b** 315

THINK

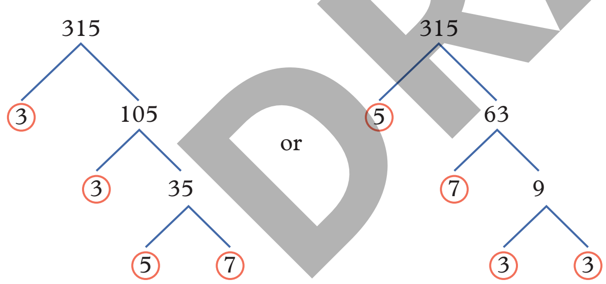
1. Identify a factor pair by dividing the composite number by its smallest prime factor. The smallest prime factor of an even number is always 2. Remember that if the sum of all the digits in a number is divisible by 3, then that number is also divisible by 3, and that any number ending in 0 or 5 is divisible by 5.
2. Continue to split factors into factor pairs until all remaining factors are prime.
3. Write the composite number as a product of its prime factors. Write the answer in index notation and list the bases in ascending order.

WRITE

a $20 = 2 \times 2 \times 5$
$= 2^2 \times 5$



b $315 = 3 \times 3 \times 5 \times 7$
$= 3^2 \times 5 \times 7$



Helpful hints

- ✔ Remember that raising a number to an index and multiplying are different operations. For example: $2^4 \neq 2 \times 4$, $2^4 = 2 \times 2 \times 2 \times 2$.
- ✔ Take care when writing indices – they should be a smaller size than the base and sit high up on the shoulder of the base to avoid confusion between 3^4 and 34.
- ✔ When creating factor trees, remember that if a branch doesn't end on a prime number, then keep dividing the composite number until the branch ends on a prime.
- ✔ Recall that the first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

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Exercise 2A Indices

1–8, 9(a, c, e, g), 10, 11, 13, 14, 17(a, b)	1–3(b, d, f, h), 4, 6–8, 9(b, d, f, h), 12, 13, 15, 17, 19	1–3(g, h), 4, 6(e–h), 8(e, f), 12–14, 16, 17(c, d), 18–20

UNDERSTANDING AND FLUENCY

2A.1 **1** Write the following in expanded form and evaluate.

a 6^4 **b** 8^3 **c** $(-2)^5$ **d** $(-3)^6$

e $\left(\frac{5}{4}\right)^3$ **f** $\left(\frac{1}{2}\right)^7$ **g** $\left(-\frac{2}{3}\right)^4$ **h** $\left(-\frac{3}{5}\right)^5$

2A.2 **2** Write the following in expanded form.

a b^6 **b** $(-n)^5$ **c** $(-cd)^2$ **d** $(2pq)^4$

e $2pq^4$ **f** $-4a^2b^3c$ **g** $(3m^2)^5$ **h** $3(m^2)^5$

3 Write the following in index notation.

a $5 \times 5 \times 5 \times 5$

b $a \times a \times a \times a$

c $v \times k \times k \times v \times k \times v \times v \times 7$

d $qu \times qu \times qu \times qu \times qu$

e $-h \times -h \times -h$

f $-(h \times h \times h)$

g $n^3 \times n^3 \times n^3 \times n^3 \times n^3 \times n^3$

h $5b^3d^4 \times 5b^3d^4$

2A.3 **4** Express each number as the product of its prime factors. Write your answers in index notation.

a 50 **b** 72 **c** 135 **d** 378

e 152 **f** 812 **g** 550 **h** 1665

5 Evaluate the following.

a $(0.2)^2$ **b** $(-0.2)^2$ **c** $(0.02)^2$

d $(0.2)^3$ **e** $(-0.2)^3$ **f** $(0.02)^3$

g $(0.2)^4$ **h** $(-0.2)^4$ **i** $(0.02)^4$

6 Write the following in index notation without brackets.

a $(-5)^4$ **b** $(-5)^3$ **c** $(ab)^4$ **d** $(5xy)^8$

e $5(xy)^8$ **f** $\left(\frac{11}{2}\right)^6$ **g** $(-3abc)^5$ **h** $(-3abc)^8$

7 Substitute in the given values and evaluate the expressions.

a x^3, where $x = 7$

b $6a^4b^2$, where $a = -2$ and $b = \frac{1}{4}$

c $\frac{p^4}{qr^3}$, where $p = 3$, $q = 5$ and $r = -4$

d $2x^3 + 8x^2 + x + 7$, where $x = 10$

8 Write the following in index notation.

a $2 \times 2 \times 2 \times 3 \times 3$

b $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 6$

c $13 \times 13 \times 13 \times 13 \times 17 \times 17 \times 17 \times 17 \times 17$

d $101 \times 101 \times 103 \times 103 \times 103 \times 103 \times 103$

e $4 \times 4 \times 4 \times x \times x \times x \times x$

f $7 \times 7 \times xy \times xy \times xy \times xy \times xy \times xy \times xy$

PROBLEM SOLVING AND REASONING

9 Use the fact that $300 = 2^2 \times 3 \times 5^2$ to help find the prime factors of each of the following numbers and then write the numbers in index notation.

a 600 b 150 c 900 d 1500

e 3000 f 1800 g 2100 h 2400

10 Express each of the following in index notation.

a $xxxxxxx$ b $aaabb$ c $3rsstt$ d $4eeeeeeeff$

11 Explain the mistake in each of the following. Then change the right-hand side so that the equation is correct.

a $tk^5 = t \times k \times t \times k \times t \times k \times t \times k \times t \times k$

b $(2rw)^4 = 2 \times r \times w \times w \times w \times w$

c $-3 \times (-2)^4 = 6^4$

12 Substitute in the given values and evaluate each expression.

a $(2x + 3)^8$, where $x = -2$

b $\left(\frac{y}{3}\right)^3 + 4\sqrt{y}$, where $y = 9$

c $ab^3 - ba^2$, where $a = 5$ and $b = -3$

d $2r^3 + 8r^2 - 3r$, where $r = -\frac{3}{2}$

13 a Evaluate each of the following.

i $(-1)^2$ ii $(-1)^3$ iii $(-1)^4$ iv $(-1)^5$ v $(-1)^6$

vi $(-1)^7$ vii $(-1)^8$ viii $(-1)^9$ ix $(-1)^{10}$ x $(-1)^{11}$

b Copy and complete the following sentences.

i When the index n is odd, the basic numeral of $(-1)^n$ is _________.

ii When the index n is even, the basic numeral of $(-1)^n$ is _________.

c Decide for each of the following whether the basic numeral will be positive or negative. Do not evaluate.

i $(-2)^{15}$ ii $(-4)^{27}$

iii $(-24)^{30}$ iv $(-17)^{198}$

v $(-16)^7 \times (-34)^{11}$ vi $(-8)^{14} \times (-5)^{27}$

vii $(-78)^{99} \times (-81)^{45} \times (-21)^{68}$ viii $\left(-\frac{77}{101}\right)^{108} \times \left(-\frac{301}{22}\right)^{404}$

14 Consider each pair of numbers written in index notation.

i Using a calculator, evaluate each pair.

ii Describe how the two numbers are similar and how they are different in their index notation and as a basic numeral.

a $(0.7)^3$ and $(0.07)^3$

b $(-0.4)^3$ and $-(0.4)^3$

c $(-1.2)^3$ and $(-1.2)^4$

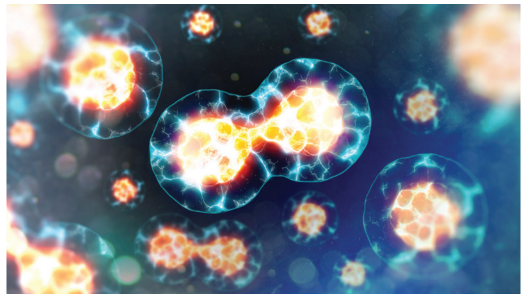
d $(2.1)^3$ and $(2.01)^3$

15 A farmer's herd of cattle grows by approximately 20% each year. In 2023, the farmer had 20 cows.

a By what number can the number of cows be multiplied to increase it by 20%?

b Predict the size of the farmer's herd in 2024, 2025 and 2028. Round your answers to the nearest whole number.

16 Three different groups of bacteria, Bacteria A, Bacteria B and Bacteria C, reproduce at different rates.



a Bacteria A splits into two bacteria every day.
How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index notation.

b Bacteria B splits into two bacteria twice each day. How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index notation.

c Bacteria C splits into two bacteria once every two days. How many times larger will a population of this bacteria be after 3, 8 and 12 days? Write your answers in index notation.

d Populations of Bacteria A, B and C each have three bacteria initially. Determine the size of each bacteria population after three days.

17 The lowest common multiple is the product of the largest index of each prime factor or pronumeral in each term. The highest common factor is the product of the smallest index of each prime factor or pronumeral in each term. Find the lowest common multiple and highest common factor of each pair of terms. Write your answers in index notation.

a $2^8 \times 3^5 \times 5^2 \times 7$ and $2^4 \times 3^{15} \times 5^2 \times 7^4$

b $a^8b^5c^2d$ and $a^4b^{15}c^2d^4$

c $pq^5r^7s^2$ and $pq^3r^{10}s^4$

d $8x^3y^9z^4$ and $12xy^3z^4$

18 For each of the following, state how many different sequences of answers there are. Write your answers in index notation.

a a quiz that has 10 true or false questions

b a quiz that has 10 multiple-choice questions each with options A, B, C, D, E

c a quiz that has 12 true or false questions and 8 multiple-choice questions with options A, B, C, D, E

19 Using positive and negative whole numbers (integers), see how many different index expressions you can find that equal 64.

$$\square^{\square} = 64$$

20 Evaluate each of the following.

a $\frac{ab^2}{c^3}$, where $a = 6$, $b = \frac{1}{3}$ and $c = -2$

b $\frac{p^3}{q^2}$, where $p = \frac{3}{2}$ and $q = \frac{2}{3}$

c $\frac{r^4}{(mn)^3}$, where $m = -0.5$, $n = 0.2$ and $r = -0.7$

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

pro

Interactive skillsheet
Indices

Investigation
Using indices to determine how card tricks work

Topic quiz
2A

2B Products and quotients of powers

Learning intentions

By the end of this topic you will be able to ...

✔ simplify products of numbers and variables with the same base

✔ simplify quotients of numbers and variables with the same base.

Inter-year link

Support	Adding and subtracting whole numbers
Year 7	1B Adding whole numbers
Year 8	4B Products and quotients of powers
Year 10	2A Indices

Product of powers law

The index laws are rules that apply to all expressions (numeric and algebraic) containing indices.

- When multiplying terms in index notation with the same base, add the indices and write the result with the same base. Writing the terms in expanded form and then simplifying achieves the same result, only at a slower pace.

  $$\begin{aligned} 2^3 \times 2^5 &= 2^{(3+5)} \\ &= 2^8 \end{aligned} \qquad \begin{aligned} a^3 \times a^5 &= a^{(3+5)} \\ &= a^8 \end{aligned}$$

  For example, $\begin{aligned} 2^3 \times 2^5 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^8 \end{aligned}$ is the same as $\begin{aligned} 2^3 \times 2^5 &= 2^{3+5} \\ &= 2^8. \end{aligned}$

- To multiply terms where variables have indices and coefficients:
  1. Multiply the coefficients of each term.
  2. Apply the product of powers law and add the indices of any common bases.
  3. Write the coefficient first, followed by the variables listed in alphabetical order.

Quotient of powers law

- When dividing terms in index notation with the same base, subtract the second index from the first index and write the result with the same base.

  $$\begin{aligned} 2^5 \div 2^3 &= 2^{(5-3)} \\ &= 2^2 \end{aligned} \qquad \begin{aligned} a^5 \div a^3 &= a^{(5-3)} \\ &= a^2 \end{aligned}$$

- Remember that quotients can be written as fractions. When simplifying fractional quotients, subtract the index of the term in the denominator from the index of the term in the numerator.

  $$\begin{aligned} \frac{2^5}{2^3} &= 2^{(5-3)} \\ &= 2^2 \end{aligned} \qquad \begin{aligned} \frac{a^5}{a^3} &= a^{(5-3)} \\ &= a^2 \end{aligned}$$

- To divide terms where variables have indices and coefficients:
  1. Divide the coefficients by their highest common factor.
  2. Apply the quotient of powers law and subtract the indices of any common bases.
  3. Write the coefficient first, followed by the variables listed in alphabetical order.

Example 2B.1 Simplifying numerical expressions using an index law

Use an appropriate index law to simplify each expression. Leave each answer in index notation.

a $3^4 \times 3^2$ **b** $7^8 \div 7^5$ **c** $\frac{8^5}{8^2}$

THINK	WRITE
a Product of powers law: add indices with a common base, 3.	**a** $3^4 \times 3^2 = 3^{4+2}$ $= 3^6$
b Quotient of powers law: subtract indices with a common base, 7.	**b** $7^8 \div 7^5 = 7^{(8-5)}$ $= 7^3$
c Remember that fractions can be written as division problems. Quotient of powers law: subtract indices with a common base, 8.	**c** $\frac{8^5}{8^2} = 8^{(5-2)}$ $= 8^3$

Example 2B.2 Using the product of powers law

Using the product of powers law, simplify each expression.

a $x^6 \times x^3$ **b** $2x^7 \times 3x^4$ **c** $a^3b^2 \times ab^{10}$

THINK	WRITE
a Product of powers law: add indices with a common base, x.	**a** $x^6 \times x^3 = x^{6+3}$ $= x^9$
b 1 Multiply the coefficients of each term. 2 Product of powers law: add indices with a common base, x. 3 Write the coefficient first, followed by the variable.	**b** $2x^7 \times 3x^4 = (2 \times 3) \times x^7 \times x^4$ $= 6 \times x^{(7+4)}$ $= 6 \times x^{11}$ $= 6x^{11}$
c 1 Product of powers law: add indices with common bases, a and b. 2 Write the variables in alphabetical order.	**c** $a^3b^2 \times ab^{10} = a^3 \times b^2 \times a^1 \times b^{10}$ $= a^{(3+1)} \times b^{(2+10)}$ $= a^4b^{12}$

Example 2B.3 Using the quotient of powers law

Using the quotient of powers law, simplify each expression.

a $x^5 \div x^2$ **b** $\frac{8x^9}{12x^5}$ **c** $a^5b^3 \div a^2b$

THINK	WRITE
a Quotient of powers law: subtract indices with a common base, x.	**a** $x^5 \div x^2 = x^{5-2}$ $= x^3$
b 1 Divide the coefficients by the highest common factor. 2 Quotient of powers law: subtract indices with a common base, x. 3 Write the coefficient first, followed by the variable.	**b** $\frac{8x^9}{12x^5} = \frac{\cancel{8}^{2} \times x^9}{\cancel{12}^{3} \times x^5}$ $= \frac{2}{3} \times \frac{x^9}{x^5}$ $= \frac{2}{3} \times x^{(9-5)}$ $= \frac{2}{3}x^4$ or $\frac{2x^4}{3}$
c 1 Quotient of powers law: subtract indices with the common bases, a and b. Remember that $b = b^1$. 2 Write the variables in alphabetical order.	**c** $a^5b^3 \div a^2b = a^{(5-2)}b^{(3-1)}$ $= a^3b^2$

Helpful hints

- ✔ 'Simplify' and 'evaluate' are different commands:
  - → To simplify in this chapter, use index laws to combine the terms and hence reduce the complexity of the calculation or numerical expression.
  - → To evaluate or 'find the value' of a calculation or numerical expression, convert the expression from index notation into a basic numeral.
- ✔ Indices only apply to the number or pronumeral immediately to the left of the index. For example, in the term $4gh^3$, the index of 3 only applies to the variable h, so $4gh^3 = 4 \times g \times h \times h \times h$.
- ✔ Recall the rules for multiplying positive and negative numbers. $+ \times - = -$
  $- \times - = +$

ANS
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Exercise 2B Products and quotients of powers

△ 1–5, 6–8(1st, 3rd columns), 9, 11(a, b), 14	□ 1(f, h, j, l), 3–9(2nd, 4th columns), 10, 11(b, c), 12(a, b), 14, 15(a–d)	⬠ 5–9(2nd, 4th columns), 10, 11(c, d), 12(b, c), 13, 15, 16

UNDERSTANDING AND FLUENCY

2B.1 **1** Use the appropriate index law to simplify each expression. Leave each answer in index notation.

a $3^5 \times 3^4$ **b** $7^8 \div 7^2$ **c** $(-2)^7 \times (-2)^5$ **d** 6×6^2

e $(-8)^{13} \div (-8)^6$ **f** $10^2 \times 10^9$ **g** $3^6 \div 3^5$ **h** $5^3 \div 5$

i $\dfrac{4^7}{4^5}$ **j** $\dfrac{(-9)^{18}}{(-9)^9}$ **k** $\dfrac{13^9}{13^6}$ **l** $2^5 \times 2^2 \times 2^3$

2 Using a calculator, calculate the basic numeral for question **1** parts **a–d**.

2B.2 **3** Using the product of powers law, simplify each expression.

a $3y^3 \times y^6$ **b** $g^2 \times 7g^5$ **c** $2b^8 \times 3b^3$ **d** $-6k^5 \times 2k^8$

e $-2b^8 \times -3b^3$ **f** $-5g^5 \times -2g \times -8g^5$ **g** $3c \times 3c^7 \times 3c^6$ **h** $p^6 \times -3p^2 \times -5p^2$

2B.3 **4** Using the quotient of powers law, simplify each expression.

a $a^6 \div a^4$ **b** $d^7 \div d^6$ **c** $g^{11} \div g$ **d** $p^{10} \div p^7$

e $a^8 \div a^3$ **f** $n^{14} \div n^{11}$ **g** $r^9 \div r$ **h** $8x^{17} \div x^6$

5 Use the product and quotient of powers laws to simplify each expression.

a $3x^5 \times 4x^6$ **b** $5x^4 \times 2x^3$ **c** $-8x^2 \times 3x^7$ **d** $-6x^{10} \times -9x$

e $6x^7 \div (2x^3)$ **f** $-20x^6 \div (-5x^2)$ **g** $4x^8 \div (10x^7)$ **h** $15x^{12} \div (9x^4)$

i $\dfrac{-24t^{18}}{3t^6}$ **j** $\dfrac{-20r^8}{-32r^2}$ **k** $\dfrac{10c^7}{2c^3}$ **l** $\dfrac{15y^{12}}{6y^5}$

6 Use the product and quotient of powers laws to simplify:

a $\dfrac{ax^{13}}{ax^4}$ **b** $\dfrac{a^2b}{a^3}$ **c** $\dfrac{m^5n}{m^5}$ **d** $\dfrac{b^{20}d}{b^{12}d}$

e $\dfrac{x^4 \times x^3}{x^2}$ **f** $\dfrac{m^7 \times m^6}{m^9}$ **g** $\dfrac{6a^2 \times a^8}{a^4}$ **h** $\dfrac{n^5 \times n^7}{n^3 \times n^4}$

i $\dfrac{5d^6 \times d^3}{d^9}$ **j** $\dfrac{8t^2 \times t^3}{-2t^3}$ **k** $\dfrac{-4k \times 3k^9}{-6k^{10}}$ **l** $\dfrac{15e^{13}}{3e^8 \times 5e^5}$

7 Simplify each expression.

a $a^3b^4 \times a^6b^2$ **b** $6m^5n^2 \times -3m^6n$ **c** $\dfrac{c^2d^9}{d^7}$ **d** $\dfrac{k^3m^8}{km^5}$

e $x^2 \times y^5 \times x^6 \times y^2$ **f** $3g^4 \times 5h^3 \times 2g^6$ **g** $a^5b^4 \times a^3b^2$ **h** $5x^6y^5 \times 3x^2y^5$

i $9w^4x^8 \times 6x^5y^4$ **j** $\dfrac{1}{tu^3} \times \dfrac{tu^7}{5}$ **k** $\dfrac{-6e^5f^{11}}{8e^4f}$ **l** $-4v^9 \times -9y^3 \times -3v^8y^7$

8 Use the product and quotient of powers laws to simplify each expression.

a $\dfrac{x^7 \times x^3}{x^4}$ **b** $\dfrac{2k^4 \times k^5}{k^6}$ **c** $\dfrac{4a^2 \times 3a^6}{2a^7}$ **d** $\dfrac{5m^2 \times 2x^4}{10x^6}$

e $\dfrac{a^5b^7 \times a^3b^6}{a^8b^{10}}$ **f** $\dfrac{n^{17}p^{13}}{n^3p^2 \times np^8}$ **g** $\dfrac{-6jq^5 \times 5j^7q^2}{15j^3q}$ **h** $\dfrac{6w^9x^6 \times 3w^4x^5}{9w^5x^4 \times w^6x^3}$

9 Use the product and quotient of powers laws to determine whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

a $x^3 \times x^4 = x^{12}$ **b** $k^3 + k^3 = k^6$ **c** $y^7 \div y = y^6$ **d** $a^5 \times a \times a^5 = a^{10}$

e $m^3n^5 \times m^2n^4 = m^{14}n^{14}$ **f** $100^8 \div 100^2 = 100^4$ **g** $\dfrac{m^3 \times m^7}{m^{11}} = \dfrac{1}{m}$ **h** $\dfrac{a^5b^6}{a^2b^4} \times \dfrac{a^3b^5}{a^4b} = a^2$

10 If the index of the denominator is greater than the index of the numerator, we can instead subtract the numerator's index from the denominator's index, leaving the base on the denominator. For example: $\dfrac{2^3}{2^5} = \dfrac{1}{2^{5-3}} = \dfrac{1}{2^2}$.

a Simplify the following. Write your answers in index notation.

i $\dfrac{3^4}{3^{10}}$ **ii** $\dfrac{5^2}{5^8}$ **iii** $\dfrac{2^5 \times 3^2}{2^9 \times 3^7}$ **iv** $\dfrac{2^5 \times 3^7}{2^9 \times 3^2}$ **v** $\dfrac{2^9 \times 3^2}{2^5 \times 3^7}$

b Copy and complete the following.

i $\dfrac{2^3}{2^5} = \dfrac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \dfrac{1}{\square \times \square} = \dfrac{1}{2^{\square}} = \dfrac{1}{2^{(\square - \square)}}$

ii $\dfrac{2^4}{2^8} = \dfrac{\square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square \times \square} = \dfrac{\square}{\square \times \square \times \square \times \square} = \dfrac{\square}{2^{\square}} = \dfrac{\square}{2^{(\square - \square)}}$

iii $\dfrac{5^6}{5^7} = \dfrac{\square \times \square \times \square \times \square \times \square \times \square}{\square \times \square \times \square \times \square \times \square \times \square \times \square} = \dfrac{\square}{\square} = \dfrac{\square}{\square^{\square}} = \dfrac{\square}{\square^{(\square - \square)}}$

11 Write the following products in index notation with prime number bases.

a $2 \times 4 \times 8 \times 16 \times 32$ **b** $3 \times 9 \times 27 \times 81 \times 243$ **c** $6 \times 36 \times 216$ **d** $4 \times 16 \times 64 \times 256 \times 1024$

12 Determine the values of the unknowns.

a $2^5 \times 3^{4x} \times 5^{12} \times 7^{z+3} = 2^w \times 3^{12} \times 5^{6y} \times 7^{11}$

b $(5^x \times 7^{4y} \times 11^z) \times (3^9 \times 5^6 \times 11) = 3^9 \times 5^{15} \times 7^{24} \times 11^5$

c $\dfrac{11^a \times 13^b \times 17^{2c} \times 19^8}{11^5 \times 13^6 \times 17^3 \times 19^d} = 11^{11} \times 13^3 \times 17^5 \times 19^2$

13 Do the product and quotient of powers laws work when the terms have different bases? Explain, using $2^4 \times 3^2$ and $y^8 \div x^5$ as examples.

14 Fill in the box to make each statement true.

a $2^{\square} = 8$ **b** $3^{\square} = 27$ **c** $5^{\square} = 25$ **d** $10^{\square} = 10\,000$

15 Fill in the box to make each statement true. Start by writing the base on the right as a power of the base on the left. For example, $8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{12}$.

a $2^{\square} = 8^4$ **b** $3^{\square} = 27^5$ **c** $5^{\square} = 25^9$ **d** $10^{\square} = 10\,000^3$

e $4^{\square} = 16^7$ **f** $2^{\square} = 32^6$ **g** $6^{\square} = 216^2$ **h** $3^{\square} = 243^6$

16 Simplify the following expressions.

a $a^mb^x \times a^nb^y$

b $a^mb^x \div (a^nb^y)$

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Products and quotients of powers

Topic quiz
2B

2C Raising indices and the zero index

Learning intentions

By the end of this topic you will be able to ...

✔ raise a term in index notation by another index
✔ evaluate calculations involving the zero index.

Inter-year links

Support	Multiplying and dividing whole numbers
Year 7	1D Multiplying whole numbers
Year 8	4C Raising indices and the zero index
Year 10	2A Indices

Power of a power law

- When raising a power to another power, multiply the indices. Writing the term in expanded form and applying the product of powers law achieves the same result, only at a slower pace.
  For example, $(2^3)^5 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{15}$ is the same as $(2^3)^5 = 2^{3\times 5} = 2^{15}$.

$$(2^3)^5 = 2^{(3\times 5)} = 2^{15} \qquad (a^2)^3 = a^{(2\times 3)} = a^6$$

- To raise an index by another index:
  1. Multiply the index of every base inside the brackets by the index outside the brackets. If there is no indicated index for a term, the index is 1 and must still be multiplied.
  2. Write the coefficient first, followed by the variables listed in alphabetical order.
- Every term inside brackets should have its index multiplied by the index outside the brackets.

$$(2\times 3)^5 = 2^5 \times 3^5 \qquad (ab)^3 = a^3 \times b^3$$

$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} \qquad \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

The zero index

- Excluding 0, any base with an index of 0 is equal to 1. This is because for every non-zero base, the index indicates the number of times we multiply 1 by the base. If we multiply 1 by the base zero times, we haven't performed any multiplications and are left with 1.

$$2^0 = 1 \qquad (-k)^0 = 1$$

- The zero index law can be demonstrated by considering the fact that any non-zero value divided by itself is equal to 1, and then applying the quotient of powers law.
  For example, $1 = \frac{a^m}{a^m} = a^{(m-m)} = a^0$.
  Therefore, $a^0 = 1$.
- The order of operations also applies to simplification. Calculations in grouping symbols should be simplified first. Remember **BIDMAS**: Brackets, Indices, Division and Multiplication, Addition and Subtraction.

Example 2C.1 Raising numerical powers by another index

Use the power of a power law to simplify the following expressions. Give your answers in index notation.

a $(3^4)^5$ **b** $(4^3 \times 7)^2$ **c** $\left(\frac{5}{2}\right)^4$ **d** $3(2^2)^4$

THINK	WRITE
a Multiply the index of the base by the index outside the brackets.	**a** $(3^4)^5 = 3^{(4 \times 5)}$ $= 3^{20}$
b Multiply the index of every base inside the brackets by the index outside the brackets. Remember that base numbers that do not have an indicated index have an index of 1, so $7 = 7^1$.	**b** $(4^3 \times 7)^2 = 4^{3 \times 2} \times 7^{1 \times 2}$ $= 4^6 \times 7^2$
c Multiply the index of every base inside the brackets by the index outside the brackets. Remember that $5 = 5^1$.	**c** $\left(\frac{5}{2}\right)^4 = \frac{5^{1 \times 4}}{2^{1 \times 4}}$ $= \frac{5^4}{2^4}$
d Multiply the index of the base by the index outside the brackets. The index outside the brackets only applies to the term inside the brackets.	**d** $3(2^2)^4 = 3 \times 2^{2 \times 4}$ $= 3 \times 2^8$

Example 2C.2 Raising algebraic powers by another index

Using the power of a power law, simplify each expression.

a $(2x^4)^3$ **b** $(3a^2b^3)^2$ **c** $\left(\frac{-x}{y^2}\right)^3$ **d** $4(a^2b)^4$

THINK	WRITE
a **1** Multiply the index of every base inside the brackets by the index outside the brackets. Remember that $2 = 2^1$.	**a** $(2x^4)^3 = 2^{1 \times 3}x^{4 \times 3}$
2 Write the coefficient first, followed by the variables listed in alphabetical order.	$= 2^3x^{12}$ or $8x^{12}$
b **1** Multiply the index of every base inside the brackets by the index outside the brackets.	**b** $(3a^2b^3)^2 = 3^{1 \times 2}a^{2 \times 2}b^{3 \times 2}$
2 Write the coefficient first, followed by the variables listed in alphabetical order.	$= 3^2a^4b^6$ or $9a^4b^6$
c Multiply the index of every base inside the brackets by the index outside the brackets. Recall that if the base is negative and the index is an odd number, then the basic numeral will be negative.	**c** $\left(\frac{-x}{y^2}\right)^3 = \frac{(-x)^{1 \times 3}}{y^{2 \times 3}}$ $= \frac{-x^3}{y^6}$ $= -\frac{x^3}{y^6}$
d **1** Multiply the index of every base inside the brackets by the index outside the brackets. The index only applies to the terms inside the brackets.	**d** $4(a^2b)^4 = 4 \times a^{2 \times 4}b^{1 \times 4}$
2 Write the coefficient first, followed by the variables listed in alphabetical order.	$= 4a^8b^4$

Example 2C.3 Simplifying expressions using the zero index

Use the property $a^0 = 1$ to simplify each expression.

a 23^0 **b** $3x^0$ **c** $(x^2)^0$

THINK	WRITE
a Any number, excluding 0, raised to an index of 0 is equal to 1.	**a** $23^0 = 1$
b Any variable raised to an index of 0 is equal to 1. Recall that an index only applies to the term immediately to its left.	**b** $3x^0 = 3 \times x^0$ $= 3 \times 1$ $= 3$
c **1** Multiply the index of every base inside the brackets by the index outside the brackets.	**c** $(x^2)^0 = x^{2 \times 0}$ $= x^0$
2 Any variable raised to the index of 0 is equal to 1.	$= 1$

Example 2C.4 Simplifying expressions using the index laws

Use the index laws to simplify each expression.

a $(x^3)^5 \times x^2$ **b** $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3}$

THINK	WRITE
a **1** Use the power of a power law to simplify the first term. Multiply the index of every base inside the brackets by the index outside the brackets.	**a** $(x^3)^5 \times x^2 = x^{3 \times 5} \times x^2$ $= x^{15} \times x^2$
2 Apply the product of powers law and add the indices of the common base, x.	$= x^{(15+2)}$ $= x^{17}$
b **1** Simplify the brackets using the power of a power law. Remember BIDMAS.	**b** $\dfrac{4x^8 \times 3x^5}{2x^4 \times (x^3)^3} = \dfrac{4x^8 \times 3x^5}{2x^4 \times x^9}$
2 Simplify the numerator and simplify the denominator.	$= \dfrac{12x^{13}}{2x^4 \times x^9}$
3 Divide the numerator by the denominator. Divide the coefficients. Keep the base and subtract the indices.	$= \dfrac{12x^{13}}{2x^{13}}$ $= 6x^0$
4 Use the property $a^0 = 1$ to simplify further.	$= 6 \times 1$ $= 6$

Helpful hints

- ✔ Take care not to mix up the index laws.
  - → across a multiplication sign, add indices $a^5 \times a^3 = a^{5+3}$
  - → across a division sign, subtract indices $a^5 \div a^3 = a^{5-3}$
  - → across brackets, multiply indices $(a^5)^3 = a^{5 \times 3}$ $(ab)^3 = a^3b^3$ $\left(\dfrac{a}{b}\right)^3 = \dfrac{a^3}{b^3}$
- ✔ Remember that $2^0 = 1$, not 0. $a^0 = 1$

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Exercise 2C Raising indices and the zero index

△ 1–10(1st, 3rd columns), 12, 13(a–e), 15(a, b)	□ 3–8(2nd, 4th columns), 9, 10(c, h, j, l), 11, 13–15, 16(b), 17	⬠ 6–8(2nd, 4th columns), 9, 10, 13, 15, 16(c, d), 18–20

UNDERSTANDING AND FLUENCY

2C.1 **1** Use the power of a power law to simplify the following expressions. Give your answer in index notation.

a $(6^4)^3$ **b** $(3^2)^2$ **c** $(3^3 \times 4)^2$ **d** $(2^6)^4$

e $(5 \times 2^7)^4$ **f** $\left(\frac{3}{4}\right)^2$ **g** $\left(\frac{5^3}{2^2}\right)^4$ **h** $\left(\frac{1}{8^5}\right)^5$

i $(-3^4)^7$ **j** $(-3^5)^4$ **k** $(-7^4 \times -11^3)^7$ **l** $\left(\frac{13^8}{-17^4}\right)^6$

2C.2 **2** Using the power of a power law, simplify each expression.

a $(b^5)^2$ **b** $(m^4)^2$ **c** $(j^5)^2$ **d** $(j^2)^5$ **e** $(n^{10})^8$ **f** $(p^{11})^9$

3 Using the power of a power law, simplify each expression. Give your answer in index notation.

a $(xy)^6$ **b** $(2d)^3$ **c** $(-5k)^7$ **d** $(9p)^{10}$

e $(-3m)^4$ **f** $\left(\frac{8}{p}\right)^2$ **g** $\left(\frac{x}{y}\right)^6$ **h** $(gh)^2$

i $(ab)^5$ **j** $\left(\frac{k}{m}\right)^3$ **k** $(-2x)^8$ **l** $\left(-\frac{d}{3}\right)^5$

4 Using the power of a power law, simplify each expression. Give your answer in index notation.

a $(3x^6)^4$ **b** $5(a^4b)^7$ **c** $\left(\frac{2m}{n}\right)^3$ **d** $\left(\frac{a^2}{b^5}\right)^4$

e $-2(u^3)^4$ **f** $\frac{4}{3}(v^7w^3)^{10}$ **g** $\frac{7}{9}\left(\frac{p}{q^6}\right)^7$ **h** $\frac{1}{2^3}(3^2r^9)^5$

i $8\left(\frac{1}{5^6t^{11}}\right)^4$ **j** $7(3i^{17})^2$ **k** $2^3(2^4c^5)^8$ **l** $\frac{5}{7}\left(\frac{5x^{25}}{7y^{30}}\right)^3$

2C.3 **5** Use the property $a^0 = 1$ to simplify each expression.

a 34^0 **b** $(18)^0$ **c** y^0 **d** $(7a)^0$

6 Use the property $a^0 = 1$ to simplify each expression.

a $2x^0$ **b** $(2x)^0$ **c** $-7y^0$ **d** $(-7y)^0$

e $-(-3c)^0$ **f** $8^0 + 4^0$ **g** $2 \times 5^0 - 3^0$ **h** $m^0 + m^0$

i $n^0 + p^0$ **j** $a^0 + b^0 + c^0$ **k** $(x + y)^0$ **l** $(-a^0)^4$

m $(5^3)^0$ **n** $(-8)^0$ **o** -8^0 **p** $-(-3)^0$

2C.4 **7** Use the index laws to simplify each expression.

a $(x^2)^4 \times x^5$ **b** $(x^5)^3 \times x^7$ **c** $x^3 \times (x^4)^6$ **d** $(x^3)^2 \times (x^7)^3$

e $\frac{x^4 \times (x^3)^5}{x^9}$ **f** $\frac{(w^2)^4 \times (w^5)^2}{(w^4)^3}$ **g** $\frac{6(b^4)^4 \times (b^3)^2}{18b^{21}}$ **h** $\frac{e^5 \times e^8}{e^3 \times e^4}$

i $\frac{(x^6)^2 \times x^3}{x^5 \times (x^2)^5}$ **j** $\frac{4a^6 \times 6(a^3)^4}{2a^4 \times 3a^5}$ **k** $\frac{t^8}{(t^2)^5} \times \frac{(t^6)^7}{t^{15}}$ **l** $(f^6)^9 \times \left(\frac{f^7}{f^2}\right)^{11}$

8 Use the index laws to simplify each expression.

a $a^3 \div a^3$ **b** $-7x^9 \div x^9$ **c** $(m^2)^3 \div m^6$ **d** $-18(b^4)^5 \div [-6(b^5)^4]$

e $y^7 \times y \div y^8$ **f** $(k^6)^0 \times k^2$ **g** $5g^4 \times 2(-g^7)^0$ **h** $3(w^5)^2 \div (w^2)^5$

i $x^8 \times (x^2)^5 \div x^3$ **j** $4p^7 \times 3p^2 \div (6p^9)$ **k** $16(b^3)^3 \div [-2(b^2)^4]$ **l** $4m^5 \times m \div [10(m^3)^2]$

9 Use the index laws to simplify each expression.

a $\frac{5(n^7)^2 \times -6(n^2)^3}{15n^2 \times (n^3)^6}$ **b** $\frac{(k^8)^2 \times k \times k^9}{k^3 \times (k^4)^2 \times k^5}$ **c** $(x^4)^2y^7 \times x^3y^2$

d $\frac{4(m^3)^4n^2 \times (m^2)^3}{8m^5n^6 \times mn}$ **e** $\frac{-3h^7k^5 \times 2h^6(k^3)^2}{(h^2)^6k^3 \times -6h(k^4)^2}$ **f** $\frac{(b^2)^2 \times ac}{a^2b \times c^3} \times bc \div a$

10 Use the index laws to simplify each expression.

a $(xy)^3 \times x^6y^4$ **b** $(2k)^5 \times (7k)^2$ **c** $(-3x^6)^4$ **d** $-5(a^4b)^7$

e $\frac{x^4}{y^5} \times \left(\frac{x}{y}\right)^6$ **f** $\left(\frac{2m}{n}\right)^3$ **g** $\left(\frac{a^2}{b^5}\right)^4$ **h** $\left(\frac{w^5x^3}{y^4}\right)^2$

i $\left(\frac{k^3m}{n^2}\right)^5 \times \left(\frac{n^3}{k^2m}\right)^4$ **j** $\left(\frac{t^4}{r^2p^3}\right)^5 \times \frac{(4r^5)^2}{p^6t^7}$ **k** $\frac{(a^3b^2)^5 \times (ab^4)^6}{(a^5b)^4}$ **l** $\frac{(3e^4)^2(2h^6)^3}{(e^2h^3)^4}$

PROBLEM SOLVING AND REASONING

11 a Simplify $a^3 \div a^3$ by first writing the expression as a fraction with each term in expanded form.

b Simplify $a^3 \div a^3$ using an index law. Leave your answer in index notation.

c Use your answers to parts **a** and **b** to explain why $a^0 = 1$.

12 Use the index laws to decide whether each statement is true or false. Explain your reasoning. For each false statement, change the right-hand side to make the statement true.

a $(3g)^4 = 3^4 \times g^4$ **b** $-8^0 = -1$ **c** $\left(\frac{x}{y}\right)^6 = \frac{x^6}{y}$ **d** $\frac{(k^3)^2 \times k^4}{k^2} = k^5$

e $6 + k^0 = 7$ **f** $100^9 \div 100^9 = 0$ **g** $\frac{m^3 \times m^8}{m^{11}} = 1$ **h** $a^0 \times b^0 = 0$

13 Find the value of x that will make each statement true.

a $2^x = 2^7$ **b** $5^x \times 5^2 = 5^6$ **c** $4^x = 1$ **d** $7^x \div 7^3 = 7^5$

e $(9^x)^2 = 9^6$ **f** $\left(\frac{2}{3}\right)^x = \frac{32}{243}$ **g** $\frac{6^x \times 6^3}{6^5} = 6^5$ **h** $(3a^x)^4 = 81a^{20}$

14 Eden simplified $3^4 \times (3^5)^3$ as $(3^9)^3 = 3^{27}$. Explain and correct her mistake.

15 The power of a power law can be explained using the product of powers law.
Complete the following.

a $(2^3)^5 = (2^3) \times (2^3) \times \underline{\quad\quad} \times \underline{\quad\quad} \times \underline{\quad\quad}$
$= 2^{3+3+\square+\square+\square}$
$= 2^{3\times\square}$

b $(x^7)^4 = (x^7) \times \underline{\quad\quad} \times \underline{\quad\quad} \times \underline{\quad\quad}$
$= x^{7+\square+\square+\square}$
$= x^{7\times\square}$

c $(2 \times 3)^4 = (2 \times 3) \times \underline{\quad\quad} \times \underline{\quad\quad} \times \underline{\quad\quad}$
$= 2 \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} \times 3 \times \underline{\quad} \times \underline{\quad} \times \underline{\quad}$
$= 2^{\square} \times 3^{\square}$

d $\left(\frac{2}{3}\right)^6 = \frac{2}{3} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square}$
$= \frac{2^{\square}}{3^{\square}}$

16 We can describe repeated addition in terms of multiplication, $2 + 2 + 2 = 2 \times 3$, and repeated multiplication in terms of raising a base to the power of an index, $2 \times 2 \times 2 = 2^3$. However, repeatedly raising a number to the same index does not require a new operation, as it can be simplified using the index laws.

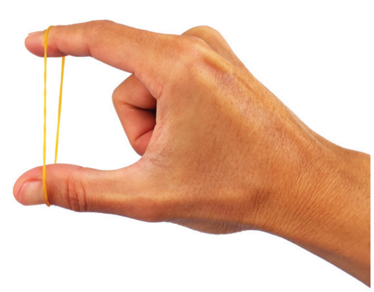
For example: $(((2^5)^5)^5) = 2^{5\times5\times5} = 2^{(5^3)} = 2^{125}$.

Write the following in index notation with a single index.

a $(((((3^2)^2)^2)^2)^2)^2$ **b** $(((5^3)^3)^3)^3$ **c** $((((7^4)^4)^4)^4)^4$ **d** $((((10^5)^5)^5)^5)^5$

17 A cube has side lengths of 8^5 cm. What is the volume of the cube in cm^3? Write your answer in index notation.

18 A rubber band is stretched to $\frac{4}{3}$ of its current length, and this is repeated another four times until it snaps. How many times longer was the rubber band when it snapped than it was originally?



CHALLENGE

19 Use the power of a power law to show that $(a^m)^n = (a^n)^m$.

20 Solve the following equation for x.

$$\frac{12x^6y^7}{7x^2y^{10}z^4} \times \frac{35y^3z^4}{3x^3} = 3$$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Raising indices and the zero index

Topic quiz
2C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

2A **1** Write the following in expanded form, then evaluate.

a 2^6 **b** $(-3)^4$ **c** $(-4)^3$ **d** $\left(\frac{5}{6}\right)^3$

2A **2** Write the following in expanded form.

a a^6 **b** $(-b)^4$ **c** $(3y)^5$ **d** $3(xy)^5$

2A **3** Write the following in index notation.

a $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$ **b** $u \times u \times u \times u$

c $4b \times 4b \times 4b \times 4b \times 4b$ **d** $-7 \times k \times k \times k \times h \times h \times h \times h \times h$

2A **4** Write the following numbers as a product of their prime factors. Express your answers in index notation.

a 28 **b** 72 **c** 484 **d** 270

2B **5** Use the index laws to simplify each expression. Express your answers in index notation.

a $8^5 \times 8^6$

b $5^7 \times 7^4 \times 5^3 \times 7^8$

c $\dfrac{6^8}{6^3}$

d $\dfrac{3^{14} \times 10^{12}}{3^6 \times 10^5}$

2B **6** Use the index laws to simplify each expression.

a $a^3 \times a^9$

b $4b^{11}c^8 \times -3b^7c^{13}$

c $\dfrac{u^{14}}{u^9}$

d $\dfrac{-15p^{17}q^{21}}{-21p^3q^4}$

2B **7** Use the index laws to simplify each expression. Write your answers in index notation.

a $\dfrac{3^7 \times 3^{12}}{3^9}$

b $\dfrac{k^{23}}{k^7 \times k^8}$

c $\dfrac{c^3t^8}{c^{14}t^7}$

d $\dfrac{9d^7w^4}{10d^{12}w^7} \times \dfrac{25d^{17}w^{12}}{6dw^2}$

2C **8** Use the index laws to simplify each expression.

a 87^0 **b** $t^5 \div t^5$ **c** $-(4g)^0$ **d** $7a^0 + (8b)^0$

2C **9** Use the index laws to simplify each expression. Write your answers in index notation.

a $(3^4)^6$ **b** $(j^5)^9$ **c** $(-5a^3b^7)^6$ **d** $-\left(\dfrac{3p^5}{2q^7}\right)^8$

2C **10** Use the index laws to simplify each expression. Write your answers in index notation.

a $\dfrac{t^4 \times (t^2)^3}{t^{10}}$

b $\dfrac{3(g^2)^8 \times (3g^5)^3}{(3g)^{11}}$

c $\dfrac{(5m^{11}n^{10})^8 \times (5mn^6)^6}{(5m^9n^7)^2 \times (5m^2n^3)^5}$

d $\dfrac{(8j^5p)^0 \times 6(j^0p^4)^3}{(j^7p^2)^6}$

2D Negative indices

Learning intentions

By the end of this topic you will be able to ...

✔ write a term with a negative index as a term with a positive index
✔ write a term with a positive index as a term with a negative index
✔ apply index laws to numerical expressions with negative indices
✔ simplify and evaluate numerical expressions with negative indices.

Inter-year links

Year 7	3F Multiplying fractions
Year 8	2C Multiplying and dividing fractions
Year 10	2B Negative indices

Reciprocals and negative indices

- The **reciprocal** of a number can be found by dividing 1 by that number.
  For example, the reciprocal of 3 is $\frac{1}{3}$.
  → The product of a number and its reciprocal is 1.
  For example, $3 \times \frac{1}{3} = 1$.
- The reciprocal of a fraction can be found by swapping the numerator with the denominator.
  For example, the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.
- A **negative index** is the reciprocal of the base with a positive index.
  For example, $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ and $4^{-2} = \frac{1}{4^2}$.
- Negative indices can be used to write fractions in index notation.
- The index laws also apply to expressions containing terms with negative indices.

Example 2D.1 Determining the reciprocals of numbers

Determine the reciprocal of each of the following.

a 3 **b** $\frac{3}{2}$ **c** $\frac{1}{4}$

THINK	WRITE
1 Write the base with a negative index as a fraction if it is not already.	**a** $3^{-1} = \frac{1}{3}$
2 Find the reciprocal of the fraction. Swap the numerator and denominator.	**b** $\left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$
3 Simplify the result.	**c** $\left(\frac{1}{4}\right)^{-1} = \frac{4}{1}$ $= 4$

Example 2D.2 Writing a term with a positive index

Write each power with a positive index.

a 3^{-3} **b** 7^{-4}

THINK	WRITE
1 Write the reciprocal of the base.	**a** $3^{-3} = \frac{1}{3^3}$
2 Change the negative index to a positive index.	**b** $7^{-4} = \frac{1}{7^4}$

Example 2D.3 Writing fractions with positive indices

Write each fraction in index notation with a positive index.

a $\frac{1}{3^{-2}}$ **b** $\left(\frac{2}{5}\right)^{-3}$

THINK	WRITE
1 Write the reciprocal of the fraction. 2 Change the negative index to a positive index.	**a** $\frac{1}{3^{-2}} = \frac{3^{-2}}{1}$ $= 3^2$
3 Use the power of a power law to remove the brackets. Recall that a number without an indicated index has an index of 1. 4 Simplify the result.	**b** $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3$ $= \frac{5^3}{2^3}$

Example 2D.4 Simplifying expressions with negative indices using index laws

Use an appropriate index law to simplify each expression. Write your answers using positive indices.

a $3^5 \times 3^{-7}$ **b** $2^4 \div 2^{-3}$ **c** $(5^{-6})^2 \times 5^3$

THINK	WRITE
a 1 Apply the product of powers law to multiply the terms. Write the base and add the indices. 2 Find the reciprocal of the fraction and write the index as a positive number.	**a** $3^5 \times 3^{-7} = 3^{(5 + (-7))}$ $= 3^{-2}$ $= \frac{1}{3^2}$
b Apply the quotient of powers law to divide the terms. Write the base and subtract the indices.	**b** $2^4 \div 2^{-3} = 2^{(4 - (-3))}$ $= 2^7$
c 1 Apply the power of a power law to simplify the first term. Multiply the index of every base inside the brackets by the index outside the brackets.	**c** $(5^{-6})^2 \times 5^3 = 5^{-6 \times 2} \times 5^3$ $= 5^{-12} \times 5^3$
2 Apply the product of powers law to multiply the terms. Write the base and add the indices.	$= 5^{(-12 + 3)}$ $= 5^{-9}$
3 Find the reciprocal of the fraction and write the index as a positive number.	$= \frac{1}{5^9}$

Helpful hints

✔ If you want to move a number or a variable from the numerator to the denominator, remember that 1 will be left in its place, not zero.

For example: $4^{-2} = \frac{1}{4^2}$.

✔ Don't confuse negative indices with negative numbers. For example: $2^{-3} = \frac{1}{2^3}$ and $2^{-3} \neq -(2^3)$.

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Exercise 2D Negative indices

1–8(a–d), 10(a–d), 11, 14, 17(a, b), 21(a, b)	1–8(e–h), 10(e–h), 12, 15, 18, 20, 21(e–h)	1(e–h), 5–6(e–h), 7–9(c–f), 10(e–h), 13, 16, 17, 19, 21–22(i–p)

UNDERSTANDING AND FLUENCY

2D.1 **1** Determine the reciprocal of each of the following.

a $\frac{8}{7}$ **b** $\frac{1}{2}$ **c** -3 **d** 9

e $-\frac{7}{3}$ **f** -9 **g** $\frac{2}{9}$ **h** 15

2D.2 **2** Write each power with a positive index.

a 5^{-1} **b** 8^{-1} **c** $(-2)^{-1}$ **d** 4^{-2}

e 2^{-6} **f** $(-9)^{-3}$ **g** $(-5)^{-4}$ **h** -7^{-8}

3 Write each of the following in index notation with a negative index.

a $\frac{1}{5}$ **b** $-\frac{1}{13}$ **c** 5 **d** -8

e $\frac{2}{7}$ **f** -21 **g** $-\frac{4}{3}$ **h** 12

4 Write each fraction in index notation with a negative index.

a $\frac{1}{3^4}$ **b** $\frac{1}{4^7}$ **c** $\frac{1}{6^5}$ **d** $\frac{1}{(-5)^3}$

e $\frac{1}{(-9)^2}$ **f** $-\frac{1}{11^6}$ **g** $-\frac{1}{4^3}$ **h** $\frac{-1}{(-6)^3}$

2D.3 **5** Write each fraction in index notation with a positive index.

a $\frac{1}{2^{-3}}$ **b** $\frac{1}{5^{-6}}$ **c** $\frac{1}{(-8)^{-4}}$ **d** $\frac{1}{3^{-9}}$

e $\frac{1}{(-7)^{-5}}$ **f** $-\frac{1}{4^{-2}}$ **g** $-\frac{1}{(-7)^{-3}}$ **h** $-\frac{1}{(-8)^{-7}}$

6 Write each fraction in index notation with a positive index.

a $\left(\frac{4}{5}\right)^{-2}$ **b** $\left(\frac{7}{3}\right)^{-1}$ **c** $\left(-\frac{3}{4}\right)^{-3}$ **d** $\left(-\frac{9}{7}\right)^{-11}$

e $\left(\frac{6}{5}\right)^{-7}$ **f** $\left(-\frac{9}{2}\right)^{-9}$ **g** $\left(-\frac{13}{17}\right)^{-14}$ **h** $\left(\frac{500}{43}\right)^{-11}$

2D.4 **7** Use an appropriate index law to simplify each expression. Write your answers in index notation with positive indices.

a $4^{-5} \times 4^2$ **b** $7^3 \times 7^{-4}$ **c** $2^{-6} \times 2^8$ **d** $(-3)^{-1} \times (-3)^{-5}$

e $5^7 \times 5^{-3}$ **f** $(-2)^{-4} \div (-2)^3$ **g** $9^5 \div 9^7$ **h** $3^6 \div 3^{-2}$

i $4^{-1} \div 4^8$ **j** $10^{-7} \div 10^{-4}$ **k** $211^{-9} \times 211^{-5}$ **l** $13^{-87} \div 13^{13}$

8 Use an appropriate index law to simplify each expression. Write your answers in index notation with positive indices.

a $(5^{-3})^2$ **b** $(3^{-2})^4$ **c** $(-2^{-4})^{-1}$ **d** $(3^{-1})^4 \times 3^2$

e $(6^{-5})^3 \times 6^{11}$ **f** $(4^{-2})^3 \times (4^{-5})^{-1}$ **g** $9^3 \times 9^{-6} \times 9^2$ **h** $\frac{5^4 \times 5^{-2}}{5^{-6}}$

i $\frac{7^{-5} \times 7^{-3}}{7^{-4} \times 7^{-7}}$ **j** $\frac{2^8 \times (2^{-2})^3}{2^5}$ **k** $\frac{(99^{-12})^{-6} \times 99^{15}}{(99^8)^{-5}}$ **l** $\frac{(15^{-9})^8 \times (15^7)^6}{(15^{11})^{12}}$

9 Using a calculator, calculate the basic numeral for parts **a–f** in question **7**. Write your answers as whole numbers or fractions.

10 Find the value of x that will make each statement true.

a $2^x = \frac{1}{2^3}$ **b** $5^x = \frac{1}{5^7}$ **c** $3^x = \frac{1}{3}$ **d** $6^x = \frac{1}{6^{-2}}$

e $4^x = \frac{1}{16}$ **f** $3^x = \frac{1}{27}$ **g** $5^x = \frac{1}{25}$ **h** $10^x = \frac{1}{10\,000}$

11 a Complete this table.

Index notation	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Basic numeral	32	16	8			1		$\frac{1}{4}$			

b Describe the pattern you can see in the table.

c Following the pattern, write 2^{-6} as a fraction.

d If 2^{10} is 1024, write the value of 2^{-10} as a fraction.

e If 2^{-7} is $\frac{1}{128}$, write the value of 2^7.

12 a Complete this table.

Index notation	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}	3^{-5}
Basic numeral		81	27			1		$\frac{1}{9}$			

b Describe the pattern you can see in the table.

c Following the pattern, write 3^{-6} as a fraction.

d If 3^8 is 6561, write the value of 3^{-8} as a fraction.

e If 3^{-7} is $\frac{1}{2187}$, write the value of 3^7.

13 a Complete this table.

Index notation	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Basic numeral			100			$\frac{1}{10}$	$\frac{1}{100}$		

b Describe the pattern you can see in the table.

c Write the value of each of the following terms as a whole number.

i 10^5 **ii** 10^6 **iii** 10^7 **iv** 10^8 **v** 10^9

d Write the value of each of the following terms as a fraction.

i 10^{-5} **ii** 10^{-6} **iii** 10^{-7} **iv** 10^{-8} **v** 10^{-9}

e Write 10^{-1} as:

i a fraction **ii** a decimal.

f Write each of the following terms as a decimal. Hint: Use the matching fractions from your table.

i 10^{-2} **ii** 10^{-3} **iii** 10^{-4}

g Write each fraction in part **d** as a decimal.

h Explain any shortcuts you used to obtain your answers to parts **c–g**.

PROBLEM SOLVING AND REASONING

14 A microscopic worm is 4^{-3} mm in length. Using a calculator, write this length in millimetres:

a as a fraction **b** as a decimal.

15 The time for light to travel 3 m is about 10^{-8} s. Using a calculator, write this time in seconds:

a as a fraction **b** as a decimal.

16 The diameter of a strand of human hair is about 5^{-6} m. Using a calculator, write this measurement in metres:

a as a fraction **b** as a decimal.

17 a Without using a calculator, find the whole number value of each of the following. Hint: What shortcut can you use when multiplying by a positive power of 10?

i 2×10^4 **ii** 7×10^3 **iii** 3×10^5

iv 4×10^{11} **v** 9×10^7

b Write each expression as a fraction involving positive indices.

i 5×10^{-2} **ii** 8×10^{-5} **iii** 2×10^{-3}

iv 7×10^{-4} **v** 6×10^{-9}

c Without using a calculator, find the decimal value of each result in part **b**. Hint: What shortcut can you use when dividing by a positive power of 10?

d Use your results from part **c** to describe a shortcut that can be used when multiplying by a negative power of 10.

18 a Complete the following by writing the missing numerals and operations.

i $2 \times 3^{-1} = 2 \times \frac{\square}{\square} = \frac{2}{\square} = 2\square 3$ **ii** $2 \div 3^{-1} = \frac{2}{\square} = 2 \times \frac{\square}{\square} = 2\square 3$

b Explain the connection between multiplication, division and reciprocals.

19 a Evaluate the following.

i $(3^{-1})^{-1}$ **ii** $(5^{-1})^{-1}$ **iii** $\left(\left(\frac{5}{4}\right)^{-1}\right)^{-1}$

b Use index laws to explain why $(a^{-1})^{-1} = a$.

c Explain what $(a^{-1})^{-1} = a$ means in terms of reciprocals.

CHALLENGE

20 Write the following as products without fractions by using negative indices.

For example: $\frac{9x^3}{y^2} = 9x^3y^{-2}$.

a $\frac{2}{a^2b}$ **b** $-\frac{3t^2}{v^3}$ **c** $\frac{x^4}{5y^4}$ **d** $\frac{1}{pq^5r^{-2}}$

21 Use an appropriate index law to simplify each expression. Write your answers using positive indices only.

a $x^4 \times x^{-6}$ **b** $x^{-3} \times x^{-1}$ **c** $4x^{-2} \times 2x^5$ **d** $5x^{-8} \times 6x^3$

e $3x^7 \times x^{-7}$ **f** $x^5 \div x^{-4}$ **g** $x^{-10} \div x^{-7}$ **h** $4x^3 \div (2x^{-2})$

i $6x^{-6} \div (18x^4)$ **j** $8x^7 \div (14x^{11})$ **k** $(x^{-2})^3 \times x^4$ **l** $(x^4)^5 \times x^{-9}$

m $(x^{-5})^3 \times 4x^2$ **n** $2x^{-3} \times (x^{-1})^5$ **o** $(x^{-4})^2 \times (x^{-3})^{-1}$ **p** $(xy)^{-7}$

22 Write all answers from question **21** that have a positive index using a negative index.

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Negative indices

Topic quiz
2D

2E Scientific notation

Learning intentions

By the end of this topic you will be able to ...

✔ convert numbers written in scientific notation to basic numerals
✔ convert numbers written as basic numerals to scientific notation.

Inter-year links

Support	Place value
Year 7	1A Place value
Year 8	1A Rounding and estimating

Scientific notation

- **Scientific notation** (or **standard form**) is a way of writing very large and very small numbers.

  $1230 = 1.23 \times 10^3$ (1230: basic numeral; 1.23×10^3: scientific notation)

- A number is written in scientific notation if it is the product of a number, a, between 1 (inclusive) and 10 (exclusive) or -1 and -10, and a power of 10, written in index notation.
  - → That is, $a \times 10^m$, where $1 \le a < 10$ or $-10 < a \le -1$ and m is an integer.

  $0.001\,23 = 1.23 \times 10^{-3}$ (0.001 23: basic numeral; 1.23×10^{-3}: scientific notation)

- When the value of a is positive:
  - → If m is a positive integer, the number is larger than or equal to 10.
  - → If m is a negative integer, the number is between 0 and 1.
  - → If m is zero, the number is between 1 and 10.
- The index laws can be used to perform operations on numbers in scientific notation.
- To convert a number in scientific notation to a basic numeral, the index indicates the number of places the decimal point is moved.
  - → If the index is positive, move the decimal point to the right.
  - → If the index is negative, move the decimal point to the left.

  10^{+} →

  10^{-} ←

- To write a number in scientific notation, place the decimal point after the first non-zero digit and multiply by the appropriate power of 10.

$$31500 = 3.15 \times 10^4 \leftarrow \text{index of 4}$$

move four spaces to the left

$$0.042 = 4.2 \times 10^{-2} \leftarrow \text{index of } -2$$

move two spaces to the right

- While we can write the expanded form of a basic numeral using powers of 10, scientific notation uses only the highest power of 10 from the expansion.

Place value	**Thousands**	**Hundreds**	**Tens**	**Ones**	**.**	**Tenths**	**Hundredths**	**Thousandths**
Index notation	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}
Basic numeral	1000	100	10	1	.	0.1	0.01	0.001

Metric prefixes

- A prefix is a letter or group of letters added to the beginning of a word to change its meaning. **Metric prefixes** can be used for very small or very large units of measurement, meaning that measurements of weight, distance and time can be expressed to a high degree of accuracy by using only a few digits.
  → Grams (g), metres (m) and seconds (s) are the base units for weight, distance and time, respectively.
- The following table details the most common metric prefixes:

Prefix	Abbreviation	Meaning	Power of 10
tera	T	one trillion of the unit	10^{12}
giga	G	one billion of the unit	10^{9}
mega	M	one million of the unit	10^{6}
kilo	K	one thousand of the unit	10^{3}
centi	c	one-hundredth of the unit	10^{-2}
milli	m	one-thousandth of the unit	10^{-3}
micro	μ	one-millionth of the unit	10^{-6}
nano	n	one-billionth of the unit	10^{-9}
pico	p	one-trillionth of the unit	10^{-12}

  → The prefixes can be added before the full names of the base units to create new units. For example, one kilogram is equal to 1000 grams.
  → Similarly, the abbreviations can be added before the abbreviated names of the base units. For example, cm is the abbreviation for centimetres, and there are 100 centimetres in a metre.

Example 2E.1 Converting numbers written in scientific notation to basic numerals

Write each number as a basic numeral.

a 2.4×10^6 **b** 7.1×10^{-8}

THINK

a Multiply by 10^6 (or 1 000 000). When multiplying by 10^6, move the decimal point six place-value spaces to the right. Add zeroes where necessary.

b Multiply by 10^{-8} (or divide by 10^8). When dividing by 10^8, move the decimal point eight place-value spaces to the left. Add zeroes where necessary.

WRITE

a 2.400000

$2.4 \times 10^6 = 2\,400\,000$

b 000000007.1

$7.1 \times 10^{-8} = 0.000\,000\,071$

Example 2E.2 Converting numbers written as basic numerals to scientific notation

Write each number in scientific notation.

a 230 000 **b** 0.000 856

THINK	WRITE
a Count the number of places the decimal point in 230 000 would be moved to produce 2.3. The decimal point needs to be moved five places to the right to obtain the original number, so the index is 5.	**a** 230000 $230\,000 = 2.3 \times 10^5$
b Count the number of places the decimal point in 0.000 856 would be moved to produce 8.56. The decimal point needs to be moved four places to the left to obtain the original number, so the index is −4.	**b** 0.000856 $0.000\,856 = 8.56 \times 10^{-4}$

Helpful hints

$10^+ \longrightarrow$

$10^- \longleftarrow$

- ✔ When converting from scientific notation to a basic numeral, remember that if the index is positive, move the decimal point to the right, and if the index is negative, move the decimal point to the left.
- ✔ Multiplying a number by 10 increases each digit's place value by one column. Move the decimal point one place-value space to the right and insert a zero where necessary.

$$5.23 \times 10 = 52.3$$

$$5.23 \times 100 = 523.$$

$$5.23 \times 1000 = 5230.$$

- ✔ Dividing a number by 10 decreases each digit's place value by one column. Move the decimal point one place-value space to the left and insert a zero where necessary.

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Exercise 2E Scientific notation

1, 2, 3(1st column), 5(a–h), 6, 7, 8(a–d), 9, 10, 12(a–c), 15, 16

3(d, g, i, k), 4, 5(e–l), 6, 8(e–h), 11, 12, 14, 19, 22

4, 5(i–p), 8(e–h), 12, 13, 17, 18, 20, 21, 23

1 Calculate each of these. Hint: Move the decimal point an appropriate number of places.

a 5.4×100 **b** $7.36 \times 10\,000$ **c** -1.8×1000

d $4.05 \times 100\,000$ **e** $2.753 \times 1\,000\,000$ **f** $\frac{6.1}{10}$

g $\frac{8.22}{1\,000\,000}$ **h** $\frac{-9.76}{10\,000}$ **i** $\frac{7.003}{100\,000}$

2 Write each number as a power of 10.

a 100 **b** 1000 **c** 10 000 **d** 100 000 **e** 1 000 000

f 0.1 **g** 0.01 **h** 0.001 **i** 0.0001 **j** 0.000 01

UNDERSTANDING AND FLUENCY

2E.1 **3** Write each number as a basic numeral.

a 3.2×10^5 **b** 8.14×10^9 **c** -5.0×10^2

d -2.345×10^7 **e** 1.1×10^4 **f** 6.4×10^{-3}

g 7.28×10^{-6} **h** 9×10^{-7} **i** -3.02×10^{-5}

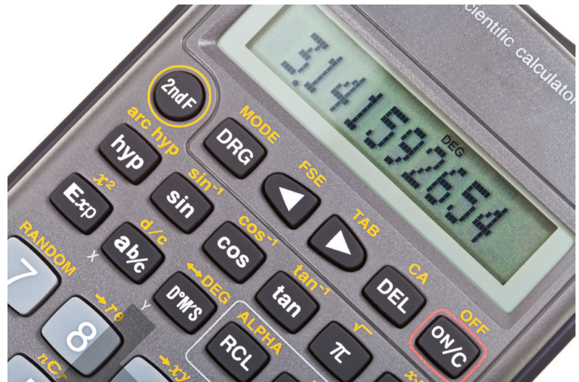
j -5.41×10^{-2} **k** 4.5×10^{11} **l** 6.12×10^{-9}

m 5.7×10^{-1} **n** 1.3068×10^3 **o** 2.7316×10^{-4}

4 Calculators use different methods for entering in and displaying numbers in scientific notation. Most scientific calculators have a button for entering numbers in scientific notation quickly. It is usually labelled with a bold **E**, Exp, $\times 10^x$ or $\times 10^n$.

Check with your teacher if you cannot find this button.

To use the button, type the value of a, press the scientific notation button, and then type the index of 10.



a Use a calculator to verify each number in question **3**.

b Were there any numbers that you could not easily display on your calculator? Explain.

2E.2 **5** Write each number in scientific notation.

a 4500 **b** 7 320 000 **c** 200 000 **d** −190

e 3216 **f** 0.0063 **g** 0.000 000 18 **h** 0.05

i −0.000 0702 **j** 0.427 **k** 11 220 **l** 0.000 004

m −568.2 **n** 0.000 249 **o** 679 300 **p** −0.0102

6 Consider the following numbers **A–H**.

A 3.4×10^4 **B** 2.03×10^{-3} **C** -0.58×10^6 **D** 60.34×10^2

E 0.009 **F** -4.19×10^3 **G** 700×10^5 **H** 9×10^{-4}

a Which numbers are written in scientific notation?

b Which numbers are larger than 10?

c Which numbers are less than 1?

7 Put the following numbers expressed in scientific notation in ascending order.

3.51×10^3, 2.814×10^4, 8.02×10^3, 7.422×10^2, 9.10×10^3, 5.76×10^2

8 Convert the following metric units into the units specified in brackets. Hint: Use the metric prefix table.

a 13 centimetres (metres) **b** 2 kilolitres (litres)

c 99 kilometres (centimetres) **d** 5 milliseconds (microseconds)

e 0.4 litres (microlitres) **f** 3000 nanoseconds (seconds)

g 0.001 gigalitres (centilitres) **h** 755 nanometres (millimetres)

9 Write each approximate measurement in scientific notation.

a A medium-sized grain of sand has a length of 0.0005 m.

b Lake Eucumbene has a capacity of approximately 4 800 000 ML.

c The thickness of the epidermal layer of skin on your eyelid is 0.048 mm.

d An estimate for the world's population in 2050 is 9 300 000 000.

10 Write each approximate measurement as a basic numeral.

a The number of times the wings of a hummingbird flap in a minute is 6.4×10^3.

b The diameter of a virus is 8×10^{-5} mm.

c The distance from the Sun to Earth is 1.496×10^8 km.

d The radius of an electron is 2.8×10^{-13} cm.

11 Complete the table below by writing the numbers as a product with each of the powers of 10. Underline the answers that are in scientific notation. The first row has been completed for you.

	1234.56	**4.0191**	**0.0492**	**0.007 40**
$\times 10^3$	$\underline{1.234\ 56 \times 10^3}$	$0.004\ 0191 \times 10^3$	$0.000\ 0492 \times 10^3$	$0.000\ 007\ 40 \times 10^3$
$\times 10^2$				
$\times 10^1$				
$\times 10^0$				
$\times 10^{-1}$				
$\times 10^{-2}$				
$\times 10^{-3}$				

12 We can perform arithmetic operations in scientific notation. Multiplication and division can be performed by multiplying or dividing the values of a and then the product and quotient of powers laws to multiply or divide the powers of 10. For example:

$$\begin{aligned}&(2.1 \times 10^7) \times (8.4 \times 10^3)\\ &= (2.1 \times 8.4) \times (10^7 \times 10^3)\\ &= 17.64 \times 10^{10}\\ &= (17.64 \div 10) \times 10^{(10+1)}\\ &= 1.764 \times 10^{11}\end{aligned}$$

$$\begin{aligned}&(2.1 \times 10^7) \div (8.4 \times 10^3)\\ &= (2.1 \div 8.4) \times (10^7 \div 10^3)\\ &= 0.25 \times 10^4\\ &= (0.25 \times 10) \times 10^{(4-1)}\\ &= 2.5 \times 10^3\end{aligned}$$

Evaluate the following products and quotients. Write your answers in scientific notation.

a $(1.7 \times 10^5) \times (4 \times 10^2)$
b $(8 \times 10^7) \div (4 \times 10^5)$
c $(-5 \times 10^{-5}) \times (-9 \times 10^8)$
d $(-6 \times 10^9) \div (1.5 \times 10^5)$
e $(4.1 \times 10^{-6}) \times (-3 \times 10^4)$
f $(7.2 \times 10^{-2}) \div (2.4 \times 10^{-7})$

13 Addition and subtraction require digits with the same place value to be added together. Therefore, in scientific notation, both numbers must be written using the same power of 10 so that the digits in the values of a have the same place value relative to the decimal point. For example:

$$\begin{aligned}&(2.1 \times 10^5) + (8.4 \times 10^3)\\ &= (2.1 \times 10^5) + (0.084 \times 10^5)\\ &= (2.1 + 0.084) \times 10^5\\ &= 2.184 \times 10^5\end{aligned}$$

$$\begin{aligned}&(2.1 \times 10^{-2}) - (8.4 \times 10^{-3})\\ &= (2.1 \times 10^{-2}) - (0.84 \times 10^{-2})\\ &= (2.1 - 0.84) \times 10^{-2}\\ &= 1.26 \times 10^{-2}\end{aligned}$$

Evaluate the following sums and differences. Write your answers in scientific notation.

a $(3.4 \times 10^2) + (7.3 \times 10^5)$
b $(8.52 \times 10^4) - (1.6 \times 10^3)$
c $(6.03 \times 10^{-3}) + (2.7 \times 10^{-4})$
d $(8.2 \times 10^{-3}) - (3.5 \times 10^{-2})$
e $(-9.8 \times 10^3) + (-7.7 \times 10^2)$
f $(1.01 \times 10^5) - (7.5 \times 10^3)$

14 Light travels at a speed of approximately 3.00×10^{10} cm/s.

a How many kilometres does it travel in 1 hour? Give your answer in scientific notation.

b How many kilometres does it travel in 1 day? Give your answer in scientific notation.

c As defined by the International Astronomical Union, a light-year is the distance light travels in 365.25 days. How far is 1 light-year in km? Give your answer in scientific notation.

15 The following table lists the mass and diameter of all the planets in the solar system.

Planet	Mass (kg)	Diameter (km)
Mercury	3.30×10^{23}	4.88×10^3
Venus	4.87×10^{24}	1.21×10^4
Earth	5.98×10^{24}	1.28×10^4
Mars	6.42×10^{23}	6.79×10^3
Jupiter	1.90×10^{27}	1.43×10^5
Saturn	5.69×10^{26}	1.21×10^5
Uranus	8.68×10^{25}	5.11×10^4
Neptune	1.02×10^{26}	4.86×10^4

a Compare the masses of the planets and list them in ascending order.

b Compare the diameters of the planets and list them in ascending order.

PROBLEM SOLVING AND REASONING

16 Earth revolves around the Sun at an average speed of 10^5 km/h.

a What distance does Earth travel in 1 day?

b How many days would it take Earth to travel 9.6×10^8 km?

17 The Sun is 1.52×10^8 km from Earth. Light from the Sun travels towards Earth at a speed of 3×10^8 m/s. How long does it take this light to reach Earth? Give your answer to the nearest minute.

18 The Australian \$1 coin has a mass of 9 g and a thickness of 3×10^{-1} cm.

a Sarah has a pile of these coins on her desk. She stacks as many of them as she can on top of each other between two shelves in a bookcase. The distance separating the shelves is 26 cm.

i How many coins are in the stack?

ii What would be the mass of these coins?

b Ben takes Sarah's stack of coins and places them end-to-end in a line. The line stretches to a length of 2.15 m. What is the diameter of a \$1 coin?

19 Write the following in seconds in scientific notation.

a 47 minutes **b** 14 days **c** 40 weeks **d** 1 year (not a leap year)

20 A number written in engineering notation is the product of a number, a, between positive or negative 1 (inclusive) and positive or negative 1000 (exclusive) and a power of 1000 written as a power of 10 in index notation. That is, $a \times 10^{3m}$, where $1 \le a < 1000$ or $-1000 < a \le -1$ and m is an integer.

For example, 3.456×10^9, 34.56×10^6, 345.6×10^{-6} are in engineering notation while 0.3456×10^9, 3456×10^6, 345.6×10^{-5} are not in engineering notation.

a Write the following in seconds in engineering notation.

i 7.3 kiloseconds (7.3 ks) **ii** 9.1 microseconds (9.1 µs)

iii 54 nanoseconds (54 ns) **iv** 82 teraseconds (82 Ts)

v 129 megaseconds (129 Ms) **vi** 974 picoseconds (974 ps)

b Write the following times in engineering notation using the appropriate prefix.

i 5.601×10^7 seconds **ii** 9.2×10^5 seconds

iii 4.31×10^{-5} seconds **iv** 7.88×10^{-7} seconds

v 8×10^{-2} seconds **vi** 1.0356×10^{14} seconds

CHALLENGE

21 Sound travels at 330 m/s, whereas light travels at 3×10^5 km/s.

a Compare the speed of light and the speed of sound.

A timekeeper stands at the end of a 100 m straight running track. The starting gun at the beginning of the track goes off.

b How long does it take:

i for the sight of the smoke to reach the timekeeper

ii for the sound of the gun to reach the timekeeper?

c What advice should you give the timekeeper in order to have an accurate recording of the time of the race?

22 The circumference of a hydrogen atom is 7.98×10^{-9} cm. How far would a line of 1 million hydrogen atoms stretch if placed next to each other?

23 Consider the multiplication problem $2^{350} \times 3^2 \times 4^3 \times 5^{355}$. Write the exact answer in scientific notation.

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Interactive skillsheet
Scientific notation

Investigation
Measuring large units in our solar system

Topic quiz
2E

2F Rounding and estimating

Learning intentions

By the end of this topic you will be able to ...

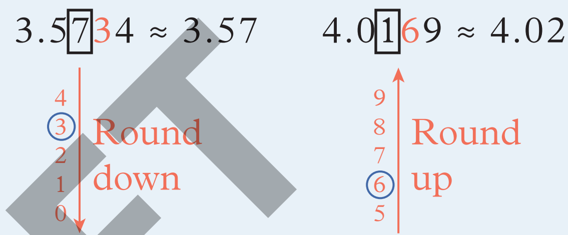
✔ round numbers to a specified degree of accuracy
✔ estimate the results of calculations
✔ determine the effect that rounding during calculations has on the accuracy of results.

Inter-year links

Year 7	4B Ordering and rounding decimals
Year 8	1A Rounding and estimating

Rounding to a given number of decimal places

- When **rounding** a number, you are replacing the number with an **approximation** that is easier to interpret and use in calculations. The approximately equal (≈) symbol should be used when rounding occurs.
- To round to a given number of decimal places, put a box around the digit that you are rounding to and look at the value of the digit to the right of the box.
  If it is 5 or greater, round up; if it is less than 5, round down.
  $3.5\boxed{7}34 \approx 3.57$ $\quad\quad$ $4.0\boxed{1}69 \approx 4.02$



Significant figures

- **Significant figures** are the number of digits required to express a number to a specified degree of accuracy.
- When counting significant figures, start by counting the first non-zero digit from left to right.
  → All non-zero digits are significant. For example, 7.789 has four significant figures, as all digits are non-zero.
  → Zeroes between two non-zero digits are significant. For example, 4056 has four significant figures including the zero between 4 and 5.
  → Leading zeroes are not significant. For example, 0.051 has two significant figures, as both the zeroes are leading zeroes.
  → Trailing zeroes to the right of the decimal point after the last non-zero significant digit are significant. For example, 112.00 has five significant figures.
  → Trailing zeroes in an integer are not significant. For example, 8300 has two significant figures.
- If a number is expressed in scientific notation, all the digits in the value of *a* are significant. For example, 2.301×10^{-2} has four significant figures.
- Rounding to a given number of significant figures involves the same process as rounding to a given number of decimal places. Put a box around the digit that you are rounding to and look at the value of the digit to the right of the box. If it is 5 or greater, round up; if it is less than 5, round down.

Truncation

- To **truncate** a number, the digits after a specified point are cut off. When truncating a number, you do not need to consider the value of the digit after the point at which you are truncating, and all the digits before that point will stay the same.
  $11.2|93 \approx 11.2$

Estimations and accuracy

- An **estimate** is an approximate value which is close to the actual value. The results of calculations can be estimated by using rounded values instead of the actual values.
- The **level of accuracy** of an approximate value describes the closeness of that approximation to the exact value. The higher the degree of accuracy, the closer the approximation is to the exact value.
  → Rounding to a greater number of decimal places or significant figures will increase the level of accuracy of an estimate.

Example 2F.1 Identifying significant figures

How many significant figures are shown in each number?

a 5.42 **b** 20 803 **c** 6.200 **d** 4000 **e** 0.0082

THINK	WRITE
a All non-zero digits are significant.	**a** 5.42 has three significant figures.
b Zeroes between non-zero digits are significant.	**b** 20 803 has five significant figures.
c Zeroes at the end of a decimal number are significant.	**c** 6.200 has four significant figures.
d Zeroes at the end of an integer are not significant.	**d** 4000 has one significant figure.
e Zeroes to the left of the first non-zero digit in a decimal number are not significant.	**e** 0.0082 has two significant figures.

Example 2F.2 Writing numbers in scientific notation using significant figures

Write each number in scientific notation with the number of significant figures indicated in brackets.

a 53 726 (2) **b** 0.084 03 (3) **c** −13.6007 (5)

THINK	WRITE
a 1 This number has five significant figures. Round to two significant figures (the nearest thousand). Remember that zeroes at the end of an integer are not significant.	**a** $53\,726 \approx 54\,000$
2 Write in scientific notation.	$= 5.4 \times 10^4$
b 1 This number has four significant figures. Round to three significant figures (the nearest ten-thousandth).	**b** $0.084\,03 \approx 0.0840$
2 Write in scientific notation. Remember that zeroes at the end of a decimal are significant.	$= 8.40 \times 10^{-2}$
c 1 This number has six significant figures. The minus sign does not impact how many significant figures a number has. Round to five significant figures (the nearest thousandth).	**c** $-13.6007 \approx -13.601$
2 Write in scientific notation.	$= -1.3601 \times 10^{-1}$

Example 2F.3 Estimating the results of calculations

Estimate the result of 535.6×38.3 by first rounding each value to two significant figures.

THINK	WRITE
1 Place a box around the digit that you are rounding to and look at the value of the digit to the right of the box. If it is 5 or greater, round up; if it is less than 5, round down.	$5\boxed{3}5.6 \approx 540$ $3\boxed{8}.3 \approx 38$
2 Complete the calculation using the rounded values.	$540 \times 38 = 20\,520$
3 Write the answer, using the approximately equal to symbol (≈).	$535.6 \times 38.3 \approx 20\,520$

Helpful hints

✔ When truncating a positive number, the truncated value will always be smaller than the original value.

✔ A truncation will never be more accurate than rounding to the same number of decimal places or significant figures.

ANS p485

Exercise 2F Rounding and estimating

△ 1–2(a–h), 3(a–d), 4–5(a–h), 6(a–d), 7(a, c, e), 10

□ 2(e–l), 3(e–h), 5(e–l), 6(e–h), 7(b, d, f, h), 8–10, 13

⬠ 3(e–h), 5(e–h), 7(c, d, g, h), 10–14

UNDERSTANDING AND FLUENCY

1 Round the following numbers to:

i one decimal place

ii two decimal places.

a 14.851	**b** 9.549	**c** 24.020	**d** 103.999
e −3.2612	**f** 17.1164	**g** −99.293	**h** 5.5454
i 72.8848	**j** −125.094	**k** 10.0030	**l** −356.8261

2F.1 2 How many significant figures are shown in each of the following numbers?

a 345	**b** 25 000	**c** 5072	**d** 400
e −809	**f** 0.59	**g** −0.003	**h** 1.472
i 48.062	**j** −7.300	**k** 36 020	**l** 0.009 04

3 How many significant figures are shown in each of the following numbers?

a 2.4×10^3	**b** 5.06×10^{-4}	**c** 1.900×10^7	**d** 8.0×10^5
e -3.206×10^{-9}	**f** 7.00×10^5	**g** -15.120×10^{-2}	**h** 220.10×10^{10}

4 Round each number to the number of significant figures indicated in brackets.

a 2.58×10^5 (2)	**b** -5.037×10^4 (3)	**c** 9.1042×10^6 (4)	**d** -6.00×10^3 (2)
e 458 (2)	**f** 73 051 (4)	**g** 1279 (1)	**h** 40 008 (1)
i −5.1437 (3)	**j** 0.0349 (2)	**k** −42.0607 (4)	**l** 0.852 (1)

UNDERSTANDING AND FLUENCY

2F.2 **5** Write each number in scientific notation with the number of significant figures indicated in brackets.

a 327 (2) **b** 48 654 (3) **c** −190 760 (4) **d** 2621 (1)

e 0.4031 (3) **f** −0.0544 (2) **g** 0.000 207 193 (4) **h** −0.008 327 (1)

i 758.4 (2) **j** −20 703.02 (4) **k** 40.155 (3) **l** 54 007.63 (5)

6 Truncate the following numbers after:

i one decimal place

ii two decimal places.

a 9.024 15 **b** 13.416 02 **c** −110.1415 **d** 80.000 01

e −4.1539 **f** −33.317 33 **g** 255.0542 **h** 1090.0148

2F.3 **7** Estimate the result of the following calculations by first rounding each value to two significant figures.

a 358.35 + 40.51 **b** 94.61 – 16.65 **c** 2450.45 + 432.91 **d** 540.67 – 249.43

e 27.98 × 11.31 **f** 123.45 ÷ 24.19 **g** 8.53 × 16.49 **h** 2984 ÷ 154.9

PROBLEM SOLVING AND REASONING

8 **a** Round each of the following to one, two and three significant figures.

i 1.901 **ii** 1.994 **iii** 1.997

iv 2.003 **v** 2.006 **vi** 2.098

b Explain how significant trailing zeroes are important in determining to how many significant figures a number is rounded.

9 Consider 0.41, 0.000 000 000 0012 and −0.000 034.

a Round each number to one significant figure.

b If leading zeroes were significant, what would each value be when rounded to one significant figure?

c Explain why not including leading zeroes as significant is more useful than including them. Consider how including them is similar to rounding to a place value or number of decimal places.

10 Explain the mistake each student made.

a Jane rounded 4.1025 to three significant figures as 4.103.

b Kaleb rounded 0.0432 to three significant figures as 0.04.

c Lisa rounded 102 948.3618 to three significant figures as 102 900.

d Marius rounded 102 948.3618 to three significant figures as 103.

11 Explain why rounding to a given number of decimal places will never give a less accurate approximation of a number when compared to truncating after the same number of decimal places.

12 Does rounding to a greater number of significant figures before performing a calculation always result in a more accurate estimate? Provide an example with your answer.

CHALLENGE

13 Convert the following measurements into the units in brackets, giving the answers in scientific notation correct to three significant figures.

a 10 000 seconds (hours) **b** 1 000 000 seconds (days)

c 1 000 000 000 seconds (years) **d** 1 000 000 000 000 seconds (millennia)

14 Find the value of the following numbers in scientific notation to two significant figures without using a calculator. Hint: $(a \times 10^b)^2 = a^2 \times 10^{2b}$.

a 2^4 **b** 2^8 **c** 2^{16} **d** 2^{32}

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Interactive skillsheet Significant figures

Investigation Investigating the accuracy of estimates

Topic quiz 2F

Chapter summary

Indices

index/exponent

base → $3^4 = 3 \times 3 \times 3 \times 3 = 81$

index notation | expanded form | basic numeral

Product of powers law

$2^3 \times 2^5 = 2^{(3+5)}$
$= 2^8$

$a^3 \times a^5 = a^{(3+5)}$
$= a^8$

Quotient of powers law

$2^5 \div 2^3 = 2^{(5-3)}$
$= 2^2$

$a^5 \div a^3 = a^{(5-3)}$
$= a^2$

Prime factorisation

315 → ③ × 105; 105 → ③ × 35; 35 → ⑤ × ⑦

$315 = 3 \times 3 \times 5 \times 7$
$= 3^2 \times 5 \times 7$

Power of a power law

$(2^3)^5 = 2^{(3\times 5)}$
$= 2^{15}$

$(a^2)^3 = a^{(2\times 3)}$
$= a^6$

$(2 \times 3)^5 = 2^5 \times 3^5$

$(ab)^3 = a^3 \times b^3$

$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$

$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

Reciprocals and negative indices

The reciprocal of 3 is $\frac{1}{3}$

The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$

$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

$4^{-2} = \frac{1}{4^2}$

The zero index

$2^0 = 1$ $a^0 = 1$

Truncation

$11.2|93 \approx 11.2$

Scientific notation

1230 = 1.23×10^3
basic numeral | scientific notation

index of 4
$3.1500 \times 10^4 = 31\,500$
move four spaces to the right

index of 4
$31500 = 3.15 \times 10^4$
move four spaces to the left

0.001 23 = 1.23×10^{-3}
basic numeral | scientific notation

index of –2
$004.2 \times 10^{-2} = 0.042$
move two spaces to the left

index of –2
$0.042 = 4.2 \times 10^{-2}$
move two spaces to the right

Rounding

$3.5\boxed{7}34 \approx 3.57$

4, ③, 2, 1, 0 — Round down

$4.0\boxed{1}69 \approx 4.02$

9, 8, 7, ⑥, 5 — Round up

Significant figures

7.789 has four significant figures
4.056 has four significant figures
0.051 has two significant figures
112.00 has five significant figures
8300 has two significant figures
2.301×10^{-2} has four significant figures

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- approximation
- base
- basic numeral
- BIDMAS
- decimal points
- estimate
- expanded form
- exponent
- factor
- factor tree
- index
- index form
- index notation
- integer
- leading zeroes
- level of accuracy
- magnitude
- metric prefix
- negative index
- place value
- positive index
- power
- prime factor
- prime factorisation
- prime number
- product
- quotient
- reciprocal
- rounding
- scientific notation
- significant figures
- standard form
- trailing zeroes
- truncate
- zero index

1 Which key term can be used to help find the prime factorisation of a number?

2 Which description best explains the term prime factorisation of a number?

A the sum of two or more integers that equals the given number

B the sum of two or more prime factors that equals the given number

C the product of two or more integers that equals the given number

D the product of two or more prime factors that equals the given number

E the product of two or more composite numbers that equals the given number

3 Use one of the numbers 7, 12 or 15 to clearly explain the difference between factors and prime factors.

4 Show how 24 can be written in expanded form and index notation.

5 Complete the following sentences using words from the key terms list.

a The _______ of a value written in _______ is the number or variable that is multiplied repeatedly.

b Taking the negative index of a fraction is the same as taking the _______ of the _______.

6 Determine if the following statements about significant figures are true or false. If they are false, give an example to support your claim.

a All non-zero digits are significant.

b All zeroes are not significant.

c All leading zeroes are not significant.

d All trailing zeroes are significant.

Multiple choice

2A **1** Which of the following is not equivalent to $9(xy)^4$?

A $9 \times xy \times xy \times xy \times xy$ **B** $-3^2x^4y^4$ **C** $9x^4y^4$

D $9xxxxyyyy$ **E** $(-3)^2x^4y^4$

2A **2** Which of the following is the prime factorisation of 360?

A $6^2 \times 10$ **B** $4 \times 9 \times 10$ **C** $2^3 \times 3^2 \times 5$ **D** $2^2 \times 3^2 \times 10$ **E** $3 \times 10 \times 12$

2B **3** Which expression shows $\frac{6ab^2c}{18a^2c}$ in simplified form?

A $\frac{6ab^2}{18a}$ **B** $\frac{ab^2c}{3a^2c}$ **C** $\frac{b^2}{3a}$ **D** $\frac{6ab^2}{18a^2}$ **E** $3ab^2$

2B **4** Which statement does not correctly represent one of the index laws?

A $m^5 \times m^2 = m^{5+2}$ **B** $(p \times q)^8 = p^8 \times q^8$ **C** $w^7 \div w^5 = w^{7-5}$

D $a^5 \times a = a^6$ **E** $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y}$

2C **5** Using the index laws, $\frac{5x^{13} \times 2x^4}{4x^8 \times x^0}$ fully simplifies to:

A $\frac{10x^9}{4x^8}$ **B** $\frac{5x^9}{2}$ **C** $\frac{5x^{17}}{2x^8}$ **D** $\frac{5}{2x^9}$ **E** $10x^9$

2D **6** Which of the following is *not* the reciprocal of $\frac{4}{3}$?

A $\frac{3}{4}$ **B** 3×4^{-1} **C** $\left(\frac{4}{3}\right)^{-1}$ **D** $\frac{1}{3 \times 4}$ **E** $1 \div \frac{4}{3}$

2D **7** Which statement is false?

A $\frac{1}{7} = 7^1$ **B** $4^{-2} = \frac{1}{16}$ **C** $\frac{1}{3^6} = 3^{-6}$ **D** $7^3 \times 7^{-5} = \frac{1}{49}$ **E** $\frac{5^{-3}}{5^4} = 5^{-7}$

2E **8** Which number is equivalent to 6.4724×10^2?

A 0.647 24 **B** 64.724 **C** 0.064 724 **D** 64 724 **E** 647.24

2F **9** What is the value of 9.1517×10^{-2} when rounded to three significant figures?

A 9.151×10^{-2} **B** 9.15×10^{-2} **C** 9.1517×10^1 **D** 9.1517×10^{-5} **E** 9.15×10^1

2F **10** What is the value of −38.725 04 when it is truncated after two decimal places?

A −38 **B** −39 **C** −38.72 **D** −38.73 **E** −38.725

Short answer

2A **1** Evaluate the following.

a 3^4 **b** $(-5)^3$ **c** -4^3 **d** $\left(\frac{3}{2}\right)^5$ **e** $(0.6)^3$ **f** $(1.2)^4$

2A **2** Write the following in index notation.

a $17 \times 17 \times 17 \times 17 \times 17 \times 17$ **b** $-5b^2 \times -5b^2 \times -5b^2 \times -5b^2 \times -5b^2$

c $-10 \times f \times f \times f \times v \times v \times v \times v \times v \times v \times v$ **d** $\frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3} \times \frac{b^4d^5}{6n^3}$

2C **3** Simplify each expression using the index laws.

a $a^{11} \times a^5$ **b** $b^9 \div b^8$ **c** $(c^8)^2$

d $18d^7 \div (54d^4)$ **e** $(e^5)^5 \times (e^{11})^2$ **f** $5a^0 + 3b^0 + 1c^0$

2C **4** Simplify each expression.

a $\frac{m^3n^4 \times m^9n^{11}}{m^7n^7}$ **b** $\frac{(3k^5l^2)^3 \times (2k^3l^3)^4}{(2k^3l^2)^3}$

2D **5** Write each term with a positive index.

a 5^{-5} **b** $(-11)^{-3}$ **c** $\frac{1}{4^{-4}}$ **d** $\left(-\frac{4}{5}\right)^{-3}$

2D **6** **a** If $4^8 = 65\,536$, write the value of 4^{-8} as a fraction.

b If $7^{-3} = \frac{1}{343}$, write the value of 7^3.

2E **7** Write each number as a basic numeral.

a 5.876×10^4 **b** 9.02×10^{-6}

2E **8** Write each number in scientific notation.

a 540 000 **b** 0.000 76

2E **9** A scientist estimates that there are 3.40×10^4 bacteria in one sample and 4.6×10^3 in a second sample. Write the total number of bacteria:

a as a basic numeral **b** in scientific notation.

2F **10** State the number of significant figures in each part of question 7.

2F **11** Round each of the following to the number of significant figures indicated in brackets.

a 879 (2) **b** 2.58×10^5 (1)

Analysis

1 Thy and Asha are playing a game. They are using die rolls and a coin flip to generate the product of three numbers in index notation per round. They each roll the dice to determine the value of the bases and indices and flip the coin to determine if each index is positive or negative.

The products generated after each round are multiplied together with the goal to end up with the least number of remaining factors after three rounds.

The table below shows the numbers Thy and Asha got in their three rounds.

	Thy	**Asha**
Round 1	$2^4 \times 4^3 \times 5^{-3}$	$1^3 \times 3^6 \times 5^{-5}$
Round 2	$3^2 \times 5^6 \times 6^{-3}$	$2^{-3} \times 3^1 \times 6^4$
Round 3	$2^{-6} \times 2^3 \times 3^4$	$3^{-5} \times 4^2 \times 5^3$

a Use the facts that $4 = 2^2$ and $6 = 2 \times 3$ to write Thy and Asha's round 1, 2 and 3 numbers in index notation with positive indices using only the bases 2, 3 and 5.

b Determine Thy and Asha's final number for their game by multiplying their round 1, 2 and 3 numbers together and simplifying the products in index notation with positive indices.

c Who won the game with the least number of factors? Hint: Find the sum of the positive indices.

d Did the winner have the smaller value? Explain.

Thy and Asha decide to play one more round of the game.

e What products do Thy and Asha need to generate to end up with a total product of 1?

Thy and Asha decide to change the rules so that they can choose which base gets which index. Their products from the first two rounds are given in the table below.

	Thy	**Asha**
Round 1	$4^{-3} \times 5^4 \times 6^5$	$2^{-3} \times 5^{-2} \times 6^4$
Round 2	$1^5 \times 2^3 \times 3^{-4}$	$3^{-3} \times 4^4 \times 5^2$

f Determine the product of round 1 and 2 for Thy and Asha. Write the products in index notation.

For round 3:

- Thy gets the bases 2, 4 and 6 and the indices −6, −4 and 1.
- Asha gets the bases 1, 5 and 5 and the indices −4, 3 and 6.

g Determine which index should go with which base so that Thy and Asha get the minimum number of factors remaining for the game.

h Who wins this game and by how many factors?

2 **a** Complete the following table.

Base (n)	n^2	n^3	n^4	n^5	n^6	n^7
3						
5						
7						
11						
13						
17						
19						

b Describe the patterns in the final digits for the different powers of each prime number in part **a**.

c Predict the last digit of n^{100} for each of the prime numbers in part **a**.

d Use the patterns you observed in part **a** to predict the last digit of $(a \times b)^{100}$ for the following products.

i 3×5 **ii** 3×7 **iii** 3×11 **iv** 7×19

v 13×17 **vi** $3^2 \times 7$ **vii** $13^2 \times 17$

3 In June 2022, the population of each Australian state was recorded. The figure for each state is shown in the table.

State	Population at 30 June 2022 ('000)
NSW	8153.6
Vic	6613.7
Qld	5322.1
SA	1820.5
WA	2785.3
Tas	571.5
NT	250.6
ACT	456.7

Source: ABS

- **a** Which states and territories have a population listed to:
  - **i** four significant figures
  - **ii** five significant figures?
- **b** Copy the table and add three additional columns.
- **c** In the first new column, write the population of each state and territory in full.
- **d** In the second new column, round each population to its leading digit.
- **e** In the third new column, write each population in scientific notation to one significant figure.
- **f** Use your answers from part **d** to determine the following. Write the values in scientific notation.
  - **i** Which state or territory has the highest population?
  - **ii** Which state or territory has the lowest population?
  - **iii** Calculate the difference between the highest and the lowest population.
  - **iv** Calculate the total population of SA, Tas, ACT and NT.
  - **v** Calculate the total population of Australia.
- **g** The actual total population value recorded at the end of June 2022 was 25 978 935.
  Calculate the difference between your answer to **f** part **v** and the actual value. Why is there a difference?

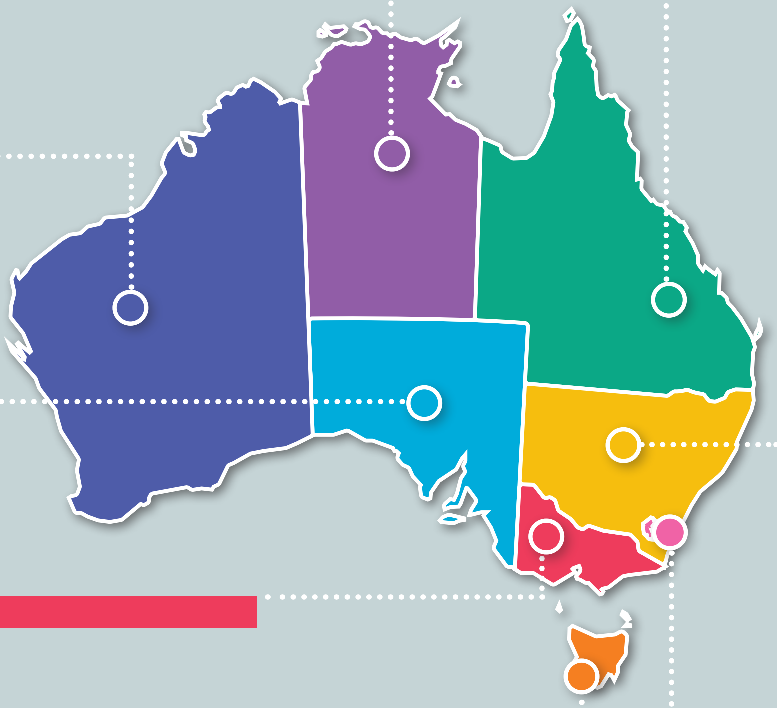
Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert between index notation and expanded form ☐ Calculate the value of numbers in index notation ☐ Express integers as a product of prime factors	☐ Go back to Topic 2A Indices
☐ Simplify products of numbers and variables with the same base ☐ Simplify quotients of numbers and variables with the same base	☐ Go back to Topic 2B Products and quotients of powers
☐ Raise a term in index notation by another index ☐ Evaluate calculations involving the zero index	☐ Go back to Topic 2C Raising indices and the zero index
☐ Write a term with a negative index as a term with a positive index ☐ Write a term with a positive index as a term with a negative index ☐ Apply index laws to numerical expressions with negative indices ☐ Simplify and evaluate numerical expressions with negative indices	☐ Go back to Topic 2D Negative indices
☐ Convert numbers written in scientific notation to basic numerals ☐ Convert numbers written as basic numerals to scientific notation	☐ Go back to Topic 2E Scientific notation
☐ Round numbers to a specified degree of accuracy ☐ Estimate the results of calculations ☐ Determine the effect that rounding during calculations has on the accuracy of results	☐ Go back to Topic 2F Rounding and estimating

Contact us

For more information about this series please contact a secondary education consultant in your state or territory:



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