MATHEMATICS

OXFORD MATHS 8 STAGE 4

SAMPLE CHAPTER

UNCORRECTED PAGE PROOFS

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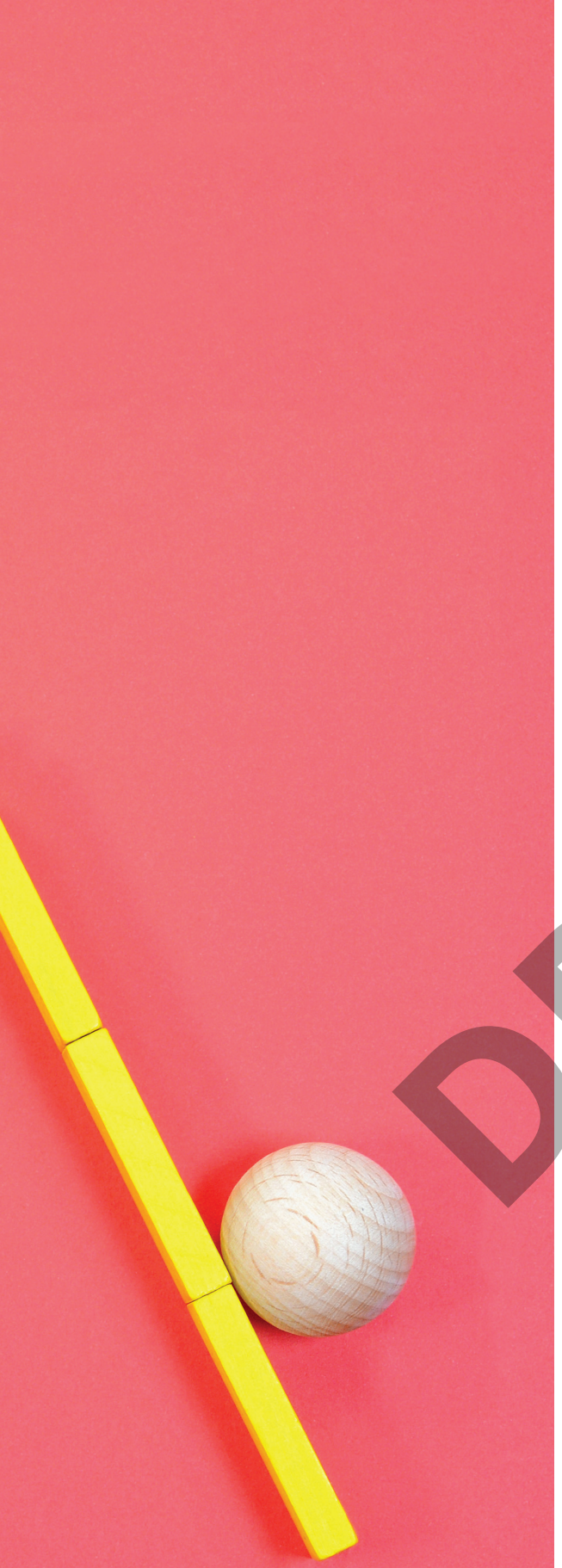
NSW CURRICULUM

1

Integers



Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Adding and subtracting two-digit numbers
- ✔ Times tables
- ✔ Multiplying and dividing by powers of 10
- ✔ Number lines

Curriculum links

- Compares, orders and calculates with integers to solve problems [MA4-INT-C-01]
  - Multiply and divide positive and negative indices
  - Apply the 4 operations to integers
- Operates with primes and roots, positive-integer and zero indices involving numerical bases and establishes the relevant index laws [MA4-IND-C-01]
  - Examine cube roots and square roots

Materials

- ✔ Calculator

1A Rounding and estimating

Learning intentions

By the end of this topic you will be able to ...

- ✔ round whole numbers
- ✔ estimate results of simple calculations
- ✔ assess the accuracy of estimations.

Inter-year links

Support	Place value
Year 7	1A Place value

Place value

- Place value charts can be used to determine the value of each digit in a number.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
3	2	8	7	2	6	3

The place value of 3 is 3 000 000
The place value of 2 is 200 000
The place value of 8 is 80 000
The place value of 7 is 7000
The place value of 2 is 200
The place value of 6 is 60
The place value of 3 is 3

→ 3 2 8 7 2 6 3

Rounding

- **Rounding** a number means to replace it with an approximation that is simpler and easier to use. Using rounded numbers makes calculations less accurate.
  - → 'Approximately equal' is shown using the symbol '≈'.
- To round to the nearest thousand, place a box around the digit in the thousands place.
  - → If the digit to the right of the box is equal to or greater than 5, **round up**.
  - → If the digit to the right of the box is less than 5, **round down**.

$12\,600 \approx 13\,000$ (9, 8, 7, (6), 5 — Round up)

$17\,300 \approx 17\,000$ (4, (3), 2, 1, 0 — Round down)

- To round to a number's **leading digit** place a box around the first digit and check if the second digit is less than, equal to or greater than 5.

Leading digit

$123\,456 = 123\,456 \approx 100\,000$

Estimations

- An **estimate** is an approximate value which is close to the actual value. We can estimate the answers of calculations by using rounding.

  For example, if 32 034 people visited Bondi Beach on Christmas Day and another 29 791 visited on Boxing Day, 32 034 + 29 791 ≈ 32 000 + 30 000 = 62 000 is an estimate of the total number of visitors. The actual number of visitors is 61 825.

Example 1A.1 Rounding a number

Write an approximation for each number by rounding to the nearest hundred.

a 17 845 **b** 4565 **c** 992

THINK

a **1** Draw a box around the digit in the hundreds place.

2 Consider the digit to the right of the boxed digit. This digit is less than 5, so do not change the boxed digit.

3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.

b **1** Draw a box around the digit in the hundreds place.

2 Consider the digit to the right of the boxed digit. This digit is greater than 5, so add one to the digit in the box.

3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.

c **1** Draw a box around the digit in the hundreds place.

2 Consider the digit to the right of the boxed digit. This digit is greater than 5, so add one to the digit in the box. The boxed digit changes from 9 to 10, so write 0 in the boxed digit's place and add one to the place to the left.

3 Replace all digits to the right of the boxed digit with a zero.

WRITE

a 17 $\boxed{8}$45
17 $\boxed{8}$45

17 $\boxed{8}$00
17 845 ≈ 17 800

b 4 $\boxed{5}$65
4 $\boxed{5}$65

4 $\boxed{6}$00
4565 ≈ 4600

c $\boxed{9}$92
$\boxed{9}$92

1 $\boxed{0}$00
992 ≈ 1000

Example 1A.2 Estimation by rounding

Estimate the result of each calculation by first rounding each number to its leading digit.

a 1307 + 4875 **b** 576 × 42 **c** 45 229 ÷ 5

THINK

1 Round each number to its leading digit.

2 Perform the calculation.

3 Write the answer using the '≈' symbol.

WRITE

a $\boxed{1}$307 + $\boxed{4}$875 ≈ 1000 + 5000
= 6000
1307 + 4875 ≈ 6000

b $\boxed{5}$76 × $\boxed{4}$2 ≈ 600 × 40
= 24 000
576 × 42 ≈ 24 000

c $\boxed{4}$5 229 ÷ $\boxed{5}$ ≈ 50 000 ÷ 5
= 10 000
45 229 ÷ 5 ≈ 10 000

Helpful hints

- ✔ Remember that the place of a digit in a number indicates its value.
  For example, the 4 in 3452 represents 400, not 4.
- ✔ The digit 0 (zero) is important. It shows that there is nothing in that place value position and it keeps all other digits in the correct places.
- ✔ Be careful when rounding to a specific place value. If you place a box around the wrong digit, your answer will not be correct. Use a place value chart if you are unsure.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1 000 000	100 000	10 000	1000	100	10	1

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Exercise 1A Rounding and estimating

 1–6, 7(a, c, e), 8(1st, 2nd columns), 9–11, 13–15, 18

 3–6, 7(b, d, f), 8(2nd, 3rd columns), 9–10(e–h), 12, 14, 16, 17, 19, 20

 3, 4, 6, 8(i–l), 10, 12, 16, 17, 20–22

UNDERSTANDING AND FLUENCY

1 Decide whether each number is closer to 400 or 500.

a 438 **b** 477 **c** 462 **d** 455 **e** 433 **f** 449

2 Decide whether each number is closer to 6000 or 7000.

a 6789 **b** 6306 **c** 6010 **d** 6505 **e** 6880 **f** 6448

1A.1 **3** **a** Round each number to the nearest ten.

i 482 **ii** 6377 **iii** 56 026 **iv** 738 494 **v** 8075 **vi** 904 507

b Round each number in part **a** to the nearest hundred.

4 **a** Round each number to the nearest thousand.

i 36 428 **ii** 7510 **iii** 183 915 **iv** 50 703 **v** 6052 **vi** 825

b Round each number in part **a** to the nearest hundred.

5 Round each number to its leading digit.

a 77 **b** 42 **c** 81 **d** 347 **e** 160

f 555 **g** 909 **h** 2489 **i** 6902 **j** 22 117

6 Round each number to its leading digit.

a 94 **b** 99 **c** 952 **d** 92 949 **e** 9008 **f** 960

7 Give three examples of numbers that round to each of these approximations if you round to the leading digit.

a 40 **b** 700 **c** 3000 **d** 50 000 **e** 800 000 **f** 2 000 000

8 Calculate:

a 900 + 700 **b** 300 − 10 **c** 30 × 20

d 80 ÷ 4 **e** 500 × 40 **f** 800 + 3000

g 200 ÷ 5 **h** 6000 − 500 **i** 40 000 ÷ 20

j 7000 × 300 **k** 20 000 − 9000 **l** 5000 + 10 000

1A.2 **9** Estimate the result of each calculation by first rounding each number to its leading digit.

a 468 + 731 **b** 92 – 38 **c** 27 × 49 **d** 83 ÷ 2

e 582 × 17 **f** 245 + 6379 **g** 4512 ÷ 43 **h** 137 – 51

10 Estimate the result of each calculation by first rounding each number to its leading digit.

a 3694 ÷ 442 **b** 187 × 9364 **c** 7085 – 750 **d** 964 + 5803

e 8277 × 65 234 **f** 13 761 + 8036 **g** 94 113 ÷ 587 **h** 24 905 – 780

11 An office building has 35 782 panes of glass. Write an approximation for this number if you round to:

a the leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten

e the nearest ten thousand.

12 Russia is the largest country in the world, with an area of 17 098 242 km^2. Round this value to:

a its leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten thousand

e the nearest hundred thousand

f the nearest million.

13 Ida is making a budget for her family's groceries. She decides to base her budget on the previous week's grocery bill, in which she spent the following:

- Supermarket $184
- Fish market $37
- Butcher $48
- Fruit and vegetable market $29

a Estimate to the nearest $10 the cost of the groceries for Ida's family last week.

b If Ida uses this figure for her budget, will it be an overestimation or underestimation?

14 Tina thinks 9648 rounded to its leading digit is 10 000. Ruby isn't sure, as she thinks the rounded number should have the same number of digits as the original number. Who is correct? Explain.

15 Daniel is saving for a quad bike. He has saved $60 per month for the past 28 months.

a Write an approximation for the number of months by rounding the value to its leading digit.

b Using your answer to part **a**, estimate the amount of money Daniel has saved.

c Does he have enough money to buy the quad bike?

d If he doesn't have enough money, estimate for how many more months he needs to save.

16 Paving tiles cost $7 each. Antonia needs 385 tiles for her back patio.

a Write an approximation of the number of tiles Antonia needs by rounding to the leading digit.

b Estimate the cost of the tiles, using your answer to part **a**.

c Compare your estimate to the exact cost of the tiles.

PROBLEM SOLVING AND REASONING

17 The school fundraising committee wants to raise money by buying boxes of sunscreen from a wholesaler to sell for a profit. Each box has a wholesale price of $47.

a Estimate the number of boxes the committee could buy with $2000.

b If the committee used this estimate, without performing the exact calculation decide whether there would be money left over or money owed to the wholesaler. Explain your answer.

18 Australia has an area of 7 692 024 km^2 and Indonesia has an area of 1 904 569 km^2.

a Round each value to its leading digit.

b Use your approximations from part **a** to estimate:

i the difference in area between the two countries

ii the area of Australia compared to the area of Indonesia.

c Check how close your estimations from part **b** are to the exact answers.

19 People attending a State of Origin rugby league match entered the stadium through one of five gates. The number of people passing through each gate is shown.

Gate	A	B	C	D	E
Number of people	9361	10 758	12 196	8844	11 037

a Estimate the number of people attending the match by rounding the numbers at each gate to the leading digit.

b Calculate the exact number of people attending the match.

20 Australia is the sixth largest country in the world. The areas of the states and territories (including islands) of Australia are shown on the map.

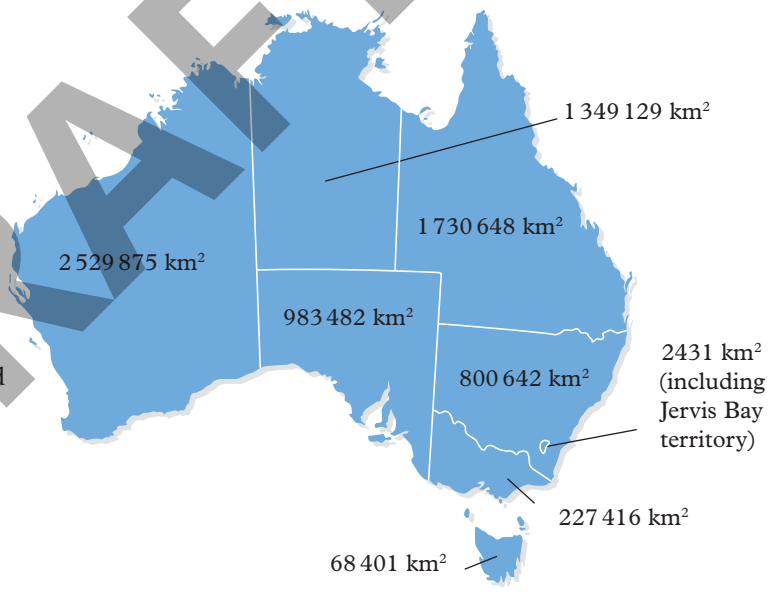
a Round each area to its leading digit.

b Estimate the area of Australia by adding up the approximate values from part **a**.

c The area of NSW, rounded to the nearest thousand, is 801 000 km^2. Round each of the other areas to the nearest thousand.

d Estimate the area of Australia by adding up the approximate values from part **c**.

e Compare the estimates you obtained in parts **b** and **d** with the accepted value for the area of Australia of 7 692 024 km^2.



CHALLENGE

21 The area of Australia is 7 692 024 km^2 and the population at the end of 2020 was 25 694 393.

a Estimate the area per person for the entire population by rounding both numbers to the leading digit.

b New Zealand has an area of 268 021 km^2 and the population at the end of 2020 was 5 106 400. Estimate the area per person for New Zealand by rounding both numbers to the leading digit and compare it with that for Australia.

22 Estimate the number of words you say in a day. Explain how you reached your estimate.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Place value
- Interactive skillsheet: Rounding integers
- Interactive skillsheet: Estimation by rounding
- Investigation: An Australian road trip
- Topic quiz: 1A

1B Adding and subtracting whole numbers

Learning intentions

By the end of this topic you will be able to ...

- ✔ use mental strategies for addition and subtraction
- ✔ use the addition and subtraction algorithms for large numbers.

Inter-year links

Support — Adding whole numbers

Year 7 — 1B Adding whole numbers

Addition properties

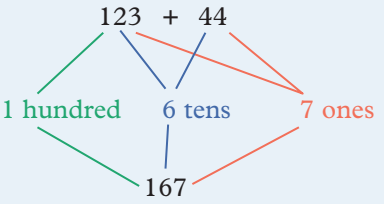
- The **associative law of addition** states that regardless of how the numbers are grouped in an addition, the result does not change.
  For example, $(3 + 5) + 2 = 3 + (5 + 2)$.
- The **commutative law of addition** states that the order in which numbers are added does not change the result.
  For example, $2 + 8 = 8 + 2$.

Addition (+)	Subtraction (−)
• Sum	• Difference
• Plus	• Take away
• More than	• Less than
• Increase	• Fewer
• Total	• Reduce
• Together	• Minus

Mental strategies

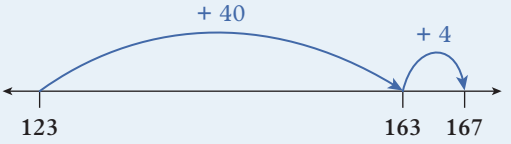
- The **by-parts method** involves adding the digits in each place value separately.

  For example, $123 + 44 = 100 + (20 + 40) + (3 + 4)$
  $= 100 + 60 + 7$
  $= 167.$



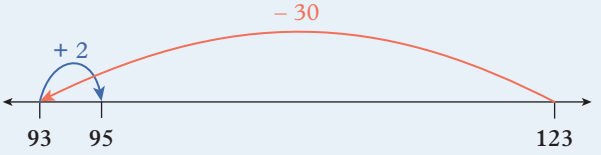
- The **jump method** involves breaking down one of the given numbers, then adding or subtracting each part of the number in stages.

  For example, $123 + 44 = 123 + 40 + 4$
  $= 163 + 4$
  $= 167.$



- The **compensation method** involves rounding one number to make the calculation easier. The amount by which the number was rounded up or down is then added or subtracted.

  For example, $123 - 28 = 123 - 30 + 2$
  $= 93 + 2$
  $= 95.$



Addition algorithm

- In the **addition algorithm**, start by adding the digits in the ones column, then add the digits in the tens column, then the hundreds column, and so on.

$$\begin{array}{r} {\scriptstyle 1} \\ 472 \\ +\ 846 \\ \hline 1318 \end{array}$$

Ones column: 2 + 6 = 8
Tens column: 7 + 4 = 11, carry the 10
Hundreds column: 4 + 8 + (1) = 13
← Answer

Subtraction algorithm

- In the **subtraction algorithm**, start by subtracting the digits in the ones column, then subtract the digits in the tens column, then the hundreds column, and so on.
  - → If the subtraction in a particular column cannot be done, take a value from the column to the left.

$$\begin{array}{r} {\scriptstyle 7\ 11} \\ 3\not{8}\not{1} \\ -\ 145 \\ \hline 236 \end{array}$$

Ones column: Take 10 from the tens column
11 − 5 = 6
Tens column: 7 − 4 = 3
Hundreds column: 3 − 1 = 2
← Answer

Example 1B.1 Using mental strategies to add two numbers

Work out each sum using the mental strategy given in brackets.

a 282 + 317 (by-parts method)
b 51 + 39 (jump method)
c 48 + 68 (compensation method)

THINK

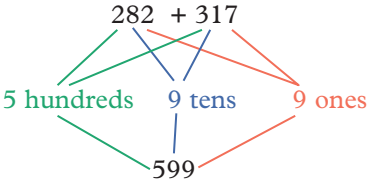
a 1 Add the hundreds together.
2 Add the tens together.
3 Add the ones together.
4 Combine these sums for the final answer.

b 1 Choose a starting number, then break down the second number into tens and ones.
2 Add the tens to the starting number, followed by the ones.

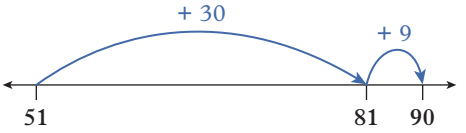
c 1 Choose a starting number, then round the second number to the nearest 10.
2 Add the rounded number to the starting number, then adjust for rounding.

WRITE

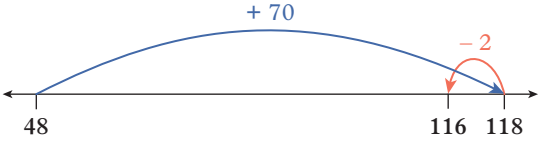
a 282 + 317 = 599



b 51 + 39 = 90



c 48 + 68 = 116



Example 1B.2 Using the addition algorithm

Calculate 781 + 965 using the addition algorithm:

$$\begin{array}{r} 781 \\ +\ 965 \\ \hline \end{array}$$

THINK

1. Add the digits in the ones column: 1 + 5 = 6.
2. Add the digits in the tens column: 8 + 6 = 14.
   Write 4 in the tens column of the answer and write 1 above the hundreds column.
3. Add the digits in the hundreds column: 1 + 7 + 9 = 17.

WRITE

$$\begin{array}{r} {\scriptstyle 1} \\ 781 \\ +\ 965 \\ \hline 1746 \end{array}$$

Example 1B.3 Using mental strategies to subtract two numbers

Work out each difference using the mental strategy given in brackets.

a 112 – 67 (jump method)

b 200 – 89 (compensation method)

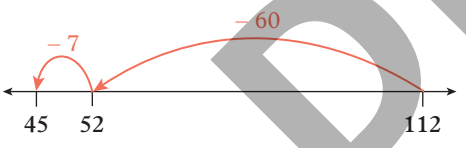
THINK

a
1. Start with the first number, then break down the second number into tens and ones.
2. Subtract the tens from that number, followed by the ones.

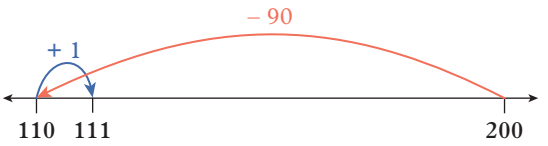
b
1. Start with the first number, then round the second number to the nearest 10.
2. Subtract the rounded number from the starting number, then adjust for rounding.

WRITE

a 112 – 67 is 45.



b 200 – 89 is 111.



Example 1B.4 Using the subtraction algorithm

Calculate 678 – 93 using the subtraction algorithm:

$$\begin{array}{r} 678 \\ -\ \ 93 \\ \hline \end{array}$$

THINK

1. Subtract the digits in the ones column: 8 – 3 = 5.
2. Subtract the digits in the tens column: 7 – 9.
   Take 1 from the hundreds column by reducing the hundreds to 5 and increasing the tens to 17.
   Subtract the digits in the tens column: 17 – 9 = 8.
3. Subtract the digits in the hundreds column: 5 – 0 = 5. Recall that if there is no digit in a place then that place has a value of zero.

WRITE

$$\begin{array}{r} {\scriptstyle 5\ 17} \\ 6\not{7}\not{8} \\ -\ \ 9\ 3 \\ \hline 5\ 8\ 5 \end{array}$$

Helpful hints

- ✔ Don't forget that addition is commutative – reordering sums can make them a lot easier!
- ✔ Subtraction is not commutative. Unlike addition problems, you cannot change the order of the numbers in subtraction problems. You must work through subtractions from left to right.
- ✔ Make sure that your place value columns are well aligned so that you add or subtract the correct digits.
- ✔ Read each question carefully. You may find some questions easy to calculate in your head.

ANS p490

Exercise 1B Adding and subtracting whole numbers

▲ 1–3, 4(a–c), 5(a, c), 6, 7, 8–10(b, c), 11, 13, 15 ■ 1a–d(ii, iii), 3(c, d), 4(d–f), 5(b, d), 6a–d(ii, iii), 8–10, 12, 14, 15, 17, 18(a) ⬟ 1a–d(iv), 4(e, f), 5(c, d), 6a–d(iv), 9(c, d), 10(c, d), 12, 14–19

UNDERSTANDING AND FLUENCY

1B.1 **1** **a** Use the by-parts method to work out each sum.

i 34 + 63 **ii** 89 + 72 **iii** 654 + 325 **iv** 1243 + 9654

b Use the jump method to work out each sum.

i 647 + 72 **ii** 768 + 51 **iii** 456 + 128 **iv** 3452 + 139

c Use the compensation method to work out each sum.

i 24 + 99 **ii** 472 + 56 **iii** 658 + 465 **iv** 7629 + 475

d Use any mental strategy to work out each sum.

i 87 + 90 **ii** 489 + 650 **iii** 8765 + 59 **iv** 5983 + 587

1B.2 **2** Calculate each sum using the addition algorithm.

a 243 + 715 **b** 538 + 154 **c** 758 + 461 **d** 697 + 805

3 Use the addition algorithm to calculate each of the following.

a 4765 + 476 **b** 7410 + 3065 **c** 16 239 + 2633 **d** 98 543 + 23 987

4 Use the addition algorithm to calculate each of the following.

a 456 + 280 + 487 **b** 876 + 387 + 594 **c** 986 + 386 + 2769

d 56 + 875 + 3987 **e** 6954 + 389 + 23 **f** 7632 + 9520 + 3198

5 Evaluate:

a 56 871 + 34 981 **b** 54 870 + 29 528 + 43 679

c 5498 + 2873 + 8921 + 7629 **d** 397 + 287 + 888 + 396 + 492

1B.3 **6** **a** Use the jump method to work out each difference.

i 93 – 30 **ii** 126 – 39 **iii** 145 – 51 **iv** 764 – 71

b Use the compensation method to work out each difference.

i 87 – 54 **ii** 763 – 68 **iii** 874 – 618 **iv** 433 – 67

c Use any mental strategy to work out each difference.

i 130 – 70 **ii** 5600 – 410 **iii** 390 – 180 **iv** 568 – 456

d Use any mental strategy to work out each problem.

i 79 + 39 – 56 **ii** 750 + 830 – 690 **iii** 78 + 565 – 120 **iv** 350 – 90 + 876

1B.4 **7** Calculate each difference using the subtraction algorithm.

a $\begin{array}{r} 438 \\ -\ 399 \\ \hline \end{array}$ **b** $\begin{array}{r} 852 \\ -\ 576 \\ \hline \end{array}$ **c** $\begin{array}{r} 2356 \\ -\ 976 \\ \hline \end{array}$ **d** $\begin{array}{r} 3765 \\ -\ 387 \\ \hline \end{array}$

8 Use the subtraction algorithm to calculate each of the following.

a 487 – 458 **b** 3865 – 376 **c** 4652 – 234 **d** 6798 – 478

9 Use the subtraction algorithm to calculate each of the following.

a 7654 – 488 **b** 7421 – 6423 **c** 75 638 – 689 **d** 86 408 – 42 766

10 Calculate:

a 566 + 238 – 328 **b** 2776 + 5409 – 3111

c 76 409 + 45 629 – 83 888 **d** 6543 – 2389 + 4319

11 Yiyan is the manager of five city car parks. The capacities of the car parks are 356, 821, 489, 398 and 450 cars. What is the total number of cars that can use Yiyan's car parks?

12 At the 2021 Census the population of NSW was 8 072 163 and the population of Greater Sydney was 5 231 147.

a How many people from NSW live outside Greater Sydney?

b How much does the population of Greater Sydney have to grow for it to become 6 000 000?

c If the population of Greater Sydney decreases by 32 000 in 2022 and increases by 59 000 in 2023, what will the population be at the end of 2023?

13 Two local libraries decide to merge their book collections to form a mega-library. Spring Hill Library has 27 356 books and Dove St Library has 54 237 books.

a How many books are there in total?

b The mega-library finds that there are two copies of 8345 books so it is decided one copy of each will be given away. After the giveaway, how many books will there be in the new library?

14 Raf and Mac go on a road trip around NSW. They combine expenses for the trip and Raf keeps a tally of the distances they travel and the money they spend.

Stage	Distance travelled (km)	Accommodation (2 nights)	Expenses (petrol, food, etc.)
Sydney to Bathurst	201	$292	$128
Bathurst to Gilgandra	262	$320	$175
Gilgandra to Narrabri	187	$228	$184
Narrabri to Muswellbrook	276	$248	$162
Muswellbrook to Sydney	251	$0	$85

a What was the total distance Raf and Mac travelled on their road trip?

b How much in total did they spend on accommodation?

c What were their total expenses not including accommodation?

d Raf and Mac had each saved up $1000 for the trip. How much was left over when they got back to Sydney?

PROBLEM SOLVING AND REASONING

15 Aya has 2437 subscribers to her YouTube channel and Akuac has 3762 subscribers.

a What is the total number of subscribers between the two girls?

b Aya and Akuac decide to combine their channels into one new channel, thus combining their subscribers.

If there are 1562 people who initially subscribed to both channels, how many subscribers will the new combined channel have? (Assuming channels can be merged and subscribers can't be counted twice.)

16 The Six Foot Track goes from Katoomba to Jenolan Caves through the Blue Mountains National Park. Jai and Rudra walk the track over 3 days in the summer. They walk the following distances: 15 km, 19 km and 10 km.

a How long is the Six Foot Track?

Katoomba is 1017 m above sea level and Jenolan Caves are 828 m above sea level.

b What is the difference in height between the two locations?

17 Shannon's shoe store has enough space for 568 pairs of shoes. If Shannon has 483 pairs of shoes in her store, how many does she need to order to be at full capacity?

CHALLENGE

18 a What is the difference between the sum of the first five odd numbers (starting from 1) and the sum of the first five even numbers (starting from 2)?

b What is the difference between the sum of the first 2000 odd numbers (starting from 1) and the first 2000 even numbers (starting from 2)?

19 A cryptarithm is a type of mathematical puzzle in which numbers have been replaced by letters. There are six solutions to the cryptarithm below. How many of them can you find?

$$\begin{array}{r} \text{SEED} \\ +\text{WATER} \\ \hline \text{FRUIT} \end{array}$$

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Adding and subtracting whole numbers
- Investigation: Palindromic numbers
- Topic quiz: 1B

1C Multiplying and dividing whole numbers

Learning intentions

By the end of this topic you will be able to ...

- ✔ use mental strategies for multiplication
- ✔ calculate products and quotients using the multiplication and division algorithms.

Inter-year links

Support	Multiplying whole numbers
Year 7	1D Multiplying whole numbers

Multiplication

- Repeated addition can be reduced to multiplication of **factors** to give a **product**.

$$\underbrace{3 + 3}_{\text{repeated addition}} = \overbrace{2 \times 3}^{\text{factors}} = \underbrace{6}_{\text{product}}$$

Multiplication strategies

- The **associative law of multiplication** states that regardless of how the numbers are grouped, the result of a multiplication does not change.

  For example, $(2 \times 4) \times 6 = 2 \times (4 \times 6)$.
- The **commutative law of multiplication** states that the order in which numbers are multiplied does not change the results.

  For example, $112 \times 4 = 4 \times 112$.
- The **distributive law** states that multiplication can be 'distributed' across a bracket. This means that large numbers can be broken down into a sum of smaller numbers, each of which can then be multiplied separately.

  For example,

$$\begin{aligned} 5 \times 32 &= 5 \times (30 + 2) \\ &= 5 \times 30 + 5 \times 2 \\ &= 150 + 10 \\ &= 160. \end{aligned}$$

Multiplication algorithm

- When two numbers are multiplied, the first number is multiplied by the individual digits of the second number, considering the place values of the digits, then the products are added together.

$$\begin{array}{rr} & {\scriptstyle 2}\,34 \\ \times & 26 \\ \hline & 204 \quad (34 \times 6) \\ + & 680 \quad (34 \times 20) \\ \hline & 884 \end{array}$$

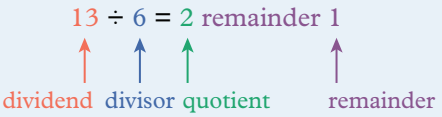
Multiply by ones → (34 × 6)

Multiply by tens → (34 × 20) Don't forget the zero(es).

Add the results →

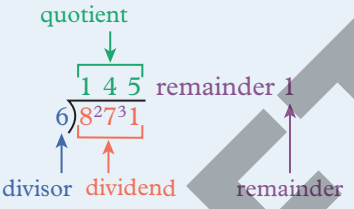
Division

- When performing a division, the **dividend** is the number that is being divided by another number.
- The **divisor** is the number by which the dividend is divided.
- The **quotient** is the whole-number result of the division.
- The **remainder** is the part of the dividend that is left over when the number is divided by the divisor. The remainder can be zero.



Short division

- **Short division** is an algorithm used to simplify a division problem by breaking it down into easy steps.
- Start by dividing the first digit of the dividend by the divisor.



Multiplication (×)	Division (÷)
• Product	• Quotient
• Times	• Divide
• Multiply	• Shared
• Groups of	• Distributed

Example 1C.1 Using strategies to multiply numbers

Work out each product using the multiplication strategy given in brackets.

a 9 × 34 (distributive law)

b 12 × 4 × 2 (associative law)

THINK

a **1** Break down the second number into tens and ones.

2 Multiply each part of the second number by the first number.

3 Add the results together.

b **1** Use brackets to group pairs of numbers that are easy to multiply together and/or produce round numbers.

2 Multiply the numbers grouped in the brackets.

3 Multiply the result by the third number.

WRITE

a $9 \times 34 = 9 \times (30 + 4)$

$= 9 \times 30 + 9 \times 4$

$= 270 + 36$

$= 306$

b $12 \times 4 \times 2 = 12 \times (4 \times 2)$

$= 12 \times 8$

$= 96$

Example 1C.2 Using the multiplication algorithm

Calculate the product using the multiplication algorithm: $\begin{array}{r} 359 \\ \times\ 72 \\ \hline \end{array}$

THINK

1. Set out the multiplication problem by lining up the digits according to their place value.
2. Multiply the first number by the ones of the second number: 359 by 2 ($359 \times 2 = 718$).
3. Multiply the first number by the tens of the second number: 359 by 70 ($359 \times 70 = 25\,130$).
4. Add the products and write the final answer at the bottom of the calculation.

WRITE

$$\begin{array}{r} {\scriptstyle 4\ 6} \\ {\scriptstyle 1\ 1} \\ 359 \\ \times\ \ 72 \\ \hline 718 \quad (359 \times 2) \\ +\ 25\,130 \quad (359 \times 70) \\ \hline 25\,848 \end{array}$$

$359 \times 72 = 25\,848$

Example 1C.3 Using short division

Use short division to calculate the quotient and remainder of $1257 \div 8$.

THINK

1. Set up the problem using the division algorithm. The divisor is 8 and the dividend is 1257.
2. How many 8s go into 1? Write a zero above the 1.
3. How many 8s go into 12? Write 1 above the 2 on the quotient line. Put the remainder 4 in front of the next digit, 5.
4. How many 8s go into 45? Write a 5 above the 5. Put the remainder 5 in front of the next digit, 7.
5. How many 8s go into 57? Write 7 above the 7 on the quotient line. Work out the remainder. $7 \times 8 = 56$, so the remainder is $57 - 56 = 1$.

WRITE

$$\begin{array}{r} 0\ 1\ 5\ 7 \text{ remainder } 1 \\ 8\overline{)1\ 2^{4}5^{5}7} \end{array}$$

$1257 \div 8 = 157$ remainder 1

Helpful hints

- ✔ When using the distributive law, don't forget to maintain place values using zeroes.
  For example, $32 \times 3 = (30 \times 3) + (2 \times 3)$
  $32 \times 3 \neq (3 \times 3) + (2 \times 3)$.
- ✔ Carefully line up digits by place value when setting up long multiplication problems to avoid making errors when adding the products together.
- ✔ Remember that division goes from left to right.

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Exercise 1C Multiplying and dividing whole numbers

▲ 1–3(1st, 2nd columns), 4(a, c), 5, 6, 7, 8–9(a–c), 10–13, 17 ■ 1–3(2nd, 3rd columns), 4(b, d), 6(c, f), 7, 8–9(d–f), 10(c, d), 11, 14, 15, 17, 19 ⬟ 1(j–l), 2(g, i), 4(d), 6(d–f), 8–9(e, f), 10(c, d), 11, 14, 16, 18–21

UNDERSTANDING AND FLUENCY

1C.1 **1** Use the distributive law to calculate:

a 3×42	**b** 6×102	**c** 203×7	**d** 305×9
e 17×98	**f** 83×99	**g** 24×95	**h** 41×19
i 62×101	**j** 29×1001	**k** 11×97	**l** 15×999

2 Use mental strategies to calculate:

a $17 \times 8 \times 3$	**b** $21 \times 5 \times 6$	**c** $3 \times 74 \times 2$
d $66 \times 8 \times 10$	**e** $4 \times 6 \times 25$	**f** $921 \times 5 \times 20$
g $35 \times 4 \times 5$	**h** $29 \times 250 \times 4$	**i** $200 \times 186 \times 5$

3 Find each product.

a 9×70	**b** 12×800	**c** 11×700	**d** 126×50
e 356×900	**f** 2765×3000	**g** 2763×9000	**h** $1674 \times 50\,000$

4 Find each product.

a $60 \times 40 \times 300$	**b** $80 \times 700 \times 8000$	**c** $2400 \times 20 \times 200$	**d** $15\,000 \times 500 \times 50$

1C.2 **5** Multiply the following pairs of numbers using the multiplication algorithm.

a 589×76	**b** 498×49	**c** 4598×23
d 4623×32	**e** 9245×72	**f** 9430×520

6 Multiply the following pairs of numbers using the multiplication algorithm.

a 387×860	**b** 378×598	**c** 638×496
d 3769×458	**e** 4287×352	**f** 5923×5030

7 **a** Estimate these products by first rounding each number to its leading digit, then multiplying.

i 8462×95	**ii** 3299×87	**iii** $95\,687 \times 625$	**iv** $78\,438 \times 8345$

b Check how close your estimations are to the exact results by performing the calculations.

1C.3 **8** Use short division to calculate each quotient and remainder.

a $8\overline{)463}$	**b** $6\overline{)348}$	**c** $7\overline{)160}$
d $5\overline{)375}$	**e** $9\overline{)689}$	**f** $4\overline{)5682}$

9 Use short division to calculate each quotient and remainder.

a $5824 \div 4$	**b** $8399 \div 6$	**c** $4210 \div 3$
d $84\,276 \div 5$	**e** $56\,289 \div 9$	**f** $1489 \div 7$

10 Perform each division using short division.

a $\frac{143}{6}$	**b** $\frac{653}{8}$	**c** $\frac{246}{7}$	**d** $\frac{489}{3}$

11 The average of a set of scores is the total sum of the scores divided by the number of scores. For example, the average of 25, 30 and 23 is $(25 + 30 + 23) \div 3$ or $\frac{25 + 30 + 23}{3}$.

a Find the sum of 19, 22, 24 and 27.

b Divide your answer from part **a** by 4 to find the average of 19, 22, 24 and 27.

12 a The commutative law relates to the order of operations. Calculate:

i $3 + 6$

ii $6 + 3$

b Does the order matter when you add two numbers? Try two more examples.

c Does subtraction obey the commutative law? Try these calculations.

i $6 - 3$

ii $3 - 6$

d Does multiplication obey the commutative law? Try these.

i 6×3

ii 3×6

e Does division obey the commutative law? Try these.

i $6 \div 3$

ii $3 \div 6$

13 a The associative law also relates to addition. Calculate:

i $(5 + 6) + 7$

ii $5 + (6 + 7)$

iii $(5 + 7) + 6$

b Does it matter in which order you add three numbers? Try two more examples.

c Do subtraction, multiplication and division obey the associative law? Try some examples to help you decide.

14 The commutative and associative laws can make mental calculations easier.

a To calculate $54 + 118 + 16$, you can add any two of the numbers together first.

i Which two numbers are easiest to add together first? Explain.

ii Perform the calculation.

b To calculate $87 \times 25 \times 4$, you can multiply any two of the numbers together first.

i Which two numbers are easiest to multiply together first? Explain.

ii Perform the calculation.

c Explain how these laws can make some calculations easier to perform.

15 a Calculate these, then compare your answers.

i $4 \times (5 + 6)$ **ii** $4 \times 5 + 4 \times 6$

b Calculate these, then compare your answers.

i $6 \times (10 - 2)$ **ii** $6 \times 10 - 6 \times 2$

c Parts **a** and **b** are examples of the application of the distributive law. Describe this law.

d Show how the distributive law works by providing two more examples.

16 The distributive law can be used to make some multiplication calculations easier to perform.

a The calculation 16×24 can be written as $16 \times (20 + 4)$ or $16 \times 20 + 16 \times 4$.

i Calculate 16×20.

ii Calculate 16×4.

iii Add your results and write the answer to 16×24.

b The calculation 45×998 can be written as $45 \times (1000 - 2)$ or $45 \times 1000 - 45 \times 2$.

i Calculate 45×1000.

ii Calculate 45×2.

iii Subtract your results and write the answer to 45×998.

c Explain how the distributive law can be used to make some multiplication calculations easier to perform.

PROBLEM SOLVING AND REASONING

17 Ms Lee spends $480 per school year on coffee from The Blue Cup Café.

a If the school year is 40 weeks, how much does Ms Lee spend per week?

b If The Blue Cup Café has a special teacher price of $4 per cup, how many times per week does Ms Lee buy coffee?

Mr Lin prefers the cheaper coffee at the service station over the road. He pays $2 per cup and buys coffee five days per week.

c How much does Mr Lin pay for coffee in one week?

d How much does Mr Lin pay for coffee in a school year?

18 The Year 8 class captains are tasked with providing icy poles for the whole school at the swimming sports carnival. There are 1289 students at the school.

a How many packs of icy poles do the class captains have to buy if there are 20 in each pack, so that no student misses out?

b How many icy poles will be left over if every student has one?

c If the packs of icy poles cost $3 each, how much money do the class captains need?

19 Astrid is planning to hire a car for a road trip from Sydney to Cairns. The distance she plans to travel is 2422 km. The car hire company informs Astrid that the car can travel an average of 7 kilometres per litre of fuel.

a Use this information to calculate how many litres of fuel Astrid will need to complete her journey.

b If Astrid's fuel tank is 50 litres, how many full tanks of fuel will she need for her journey?

c Astrid wants to make sure she overestimates the amount of money she will need to spend on fuel, so she rounds the price of fuel up to $2 per litre. Using this estimate, how much money should Astrid budget for her fuel expenses?

CHALLENGE

20 Darcy makes a number of mini spring rolls. After eating one, she gives half the remainder to her sister. After eating another spring roll, she gives half of what is left to her brother. Darcy now has only five spring rolls left. How many did she start with?

21 A recipe for eight biscuits requires 20 g butter, 30 g sugar and 40 g rolled oats.

a How many grams of sugar do I need if I want to make 16 biscuits?

b How many biscuits can I make if I have 140 g butter, 150 g sugar and 160 g rolled oats?

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Multiplying and dividing whole numbers

Investigation
Factorials

Topic quiz
1C

1D Multiples, factors, indices and roots

Learning intentions

By the end of this topic you will be able to ...

- ✔ find the lowest common multiple and highest common factor of two numbers
- ✔ evaluate squares and cubes of numbers
- ✔ evaluate square roots and cube roots.

Inter-year links

Support	Multiples and factors
Year 7	2C Multiples and the lowest common multiple

Multiples

- The **multiples** of a whole number are the numbers produced by multiplying that number by another whole number.
  For example, some multiples of 3 are:
  $3 \times 1 = 3$
  $3 \times 2 = 6$
  $3 \times 3 = 9$.
- The **lowest common multiple (LCM)** of two or more numbers is the common multiple of the numbers with the smallest value.

Multiples of 4: 4, 8, (12), 16, ...
Multiples of 6: 6, (12), 18, 24, ...
LCM: 12

Factors

- The factors of a whole number are the whole numbers which divide exactly into that number.
  For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- The divisibility rules are helpful when looking for factors of large numbers.

÷ 2 The last digit of the number is even.	÷ 3 The sum of all the digits in the number is divisible by 3.	÷ 4 The number made by the last two digits is divisible by 4.
÷ 5 The last digit is 5 or 0.	÷ 6 The number is divisible by 2 and 3.	÷ 7 There is no easy trick for 7.
÷ 8 The number made by the last three digits is divisible by 8.	÷ 9 The sum of all the digits in the number is divisible by 9.	÷ 10 The last digit is 0.

- The **highest common factor (HCF)** of two or more whole numbers is the **common factor** of the numbers with the greatest value.

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 18: 1, 2, 3, 6, 9, 18
HCF: 6

Indices

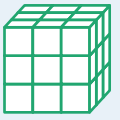
- Repeated multiplications can be represented using indices.

index
base → $2^3 = 2 \times 2 \times 2 = 8$
index form | expanded form | basic numeral

- The **base** is the number that is repeatedly multiplied.
- The **index** (or **exponent**) represents the number of times the base appears in **expanded form**.

Squares, cubes and roots

- The **square** of a number is the result of multiplying that number by itself. Squaring a number is the same as raising it to a power (or index) of 2 and is indicated by a superscript 2.
  For example, $5^2 = 5 \times 5 = 25$.
- The **square root** of a number is the number that when squared results in the original number. Square roots are indicated using the symbol $\sqrt{\ }$.
  For example, $\sqrt{9} = 3$ because $3^2 = 3 \times 3 = 9$.
- For positive numbers, there is an inverse relationship between squares and square roots.
  This means that if $8^2 = 64$, then $\sqrt{64} = 8$.
- The **cube** of a number is the result of multiplying that number by itself twice. Cubing a number is the same as raising it to a power (or index) of 3 and is indicated by a superscript 3.
  For example, $4^3 = 4 \times 4 \times 4 = 64$.
- The **cube root** of a number is the number that when cubed results in the original number. Cube roots are indicated using the symbol $\sqrt[3]{\ }$.
  For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$.
- There is an inverse relationship between cubes and cube roots.
  This means that if $5^3 = 125$, then $\sqrt[3]{125} = 5$.
- When multiplying a number by a square or cube root, we can write the number in front of the root without using a multiplication sign.
  For example, $5\sqrt{9}$ means the same as $5 \times \sqrt{9}$.

Squares			Cubes		
$1^2 = 1 \times 1 = 1$		$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$		$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$		$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$		$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$		$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$		$\sqrt[3]{27} = 3$

Example 1D.1 Finding the LCM and HCF

Determine:

a the LCM of 14 and 35

b the HCF of 24 and 38.

THINK

a **1** List the multiples of 14 and the multiples of 35.

2 Draw a box around the multiple with the lowest value that appears in both lists (the LCM).

b **1** List all the factors of 24 and all the factors of 38.

2 Draw a box around the factor with the highest value that appears in both lists (the HCF).

WRITE

a Multiples of 14: 14, 28, 42, 56, 70, …
Multiples of 35: 35, 70, 105, 140, …
The LCM is 70.

b Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 38: 1, 2, 19, 38
The HCF is 2.

Example 1D.2 Calculating the square and cube

Calculate:

a the square of 5

b the cube of 5.

THINK

1 To 'square' 5, raise 5 to the power of 2. To 'cube' 5, raise 5 to the power of 3.

2 Identify the base and the index and multiply the base by itself as many times as specified by the index.

3 Perform the multiplication.

WRITE

a $5^2 = 5 \times 5$
$= 25$

b $5^3 = 5 \times 5 \times 5$
$= (5 \times 5) \times 5$
$= 25 \times 5$
$= 125$

Example 1D.3 Calculating the square root and cube root

Calculate the value of each of the following roots.

a $\sqrt{36}$

b $\sqrt[3]{64}$

THINK

a Use times tables to determine which number, multiplied by itself, is equal to the number under the square root symbol.

b Use times tables and estimation to determine which number, multiplied by itself three times, is equal to the number under the cube root symbol.

WRITE

a $6 \times 6 = 36$
So, $\sqrt{36} = 6$

b $4^3 = 4 \times 4 \times 4$
$= (4 \times 4) \times 4$
$= 16 \times 4$
$= 64$
So, $\sqrt[3]{64} = 4$

Helpful hints

- ✔ Lowest common multiples are not necessarily 'low' in value; they can be large numbers.
- ✔ There is no such thing as a 'highest common multiple', as any given number has infinitely many multiples.
- ✔ Make sure you use a systematic method for finding all the factors; it is very easy to miss one or two.
- ✔ Don't confuse indices and multiplication!
  For example, $2^3 = 2 \times 2 \times 2$
  $2^3 \neq 2 \times 3$.
  Place your indices carefully; they should be in smaller font than the base and sit high up on the shoulder of the base – your 4^3 should look different to your 43!

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Exercise 1D Multiples, factors, indices and roots

 1–5, 6(a, c, e), 7, 8(a–f), 9(1st, 2nd columns), 10, 11, 16, 17

 2, 3, 6, 7, 8(g–l), 9(3rd, 4th columns), 10–15, 17

 3(d–f), 6(b, d, f), 7, 8(j, k, l), 9(3rd, 4th columns), 10, 11, 15, 17–20

1 **a** List the multiples of 6 and 9 up to 100.
b Identify the common multiples of 6 and 9 in the lists from part **a**.
c What is the lowest common multiple (LCM) of 6 and 9?

1D.1 **2** Find the lowest common multiple (LCM) of each pair of numbers.

a 6 and 8 **b** 12 and 18
c 25 and 30 **d** 12 and 15
e 14 and 20 **f** 9 and 15

3 Find the lowest common multiple (LCM) of each group of numbers.

a 5, 6 and 7 **b** 5, 7 and 9
c 6, 8 and 9 **d** 5, 8 and 12
e 15, 18 and 24 **f** 9, 18 and 27

4 Find all the factors of each number.

a 45 **b** 72 **c** 90
d 120 **e** 100 **f** 144

5 **a** What is the highest common factor (HCF) of 18 and 36?
b What is the HCF of 21 and 45?
c What is the HCF of 12 and 30?

6 Find the highest common factor (HCF) of each group of numbers.

a 42 and 70 **b** 48 and 84
c 36 and 63 **d** 144, 180 and 200
e 120, 240 and 360 **f** 500, 900 and 725

7 Find the LCM and the HCF of each group of numbers.

a 24 and 45 **b** 16 and 36 **c** 6, 15 and 20 **d** 7, 12 and 22

1D.2 **8** Calculate the square and the cube of each of the following numbers.

a 3 **b** 2 **c** 10 **d** 7 **e** 6 **f** 12
g 8 **h** 4 **i** 1 **j** 9 **k** 11 **l** 14

1D.3 **9** Calculate the value of each of the following roots.

a $\sqrt{9}$	**b** $\sqrt[3]{8}$	**c** $\sqrt[3]{125}$	**d** $\sqrt{64}$
e $\sqrt{81}$	**f** $\sqrt[3]{216}$	**g** $\sqrt{100}$	**h** $\sqrt[3]{343}$
i $\sqrt[3]{512}$	**j** $\sqrt[3]{2197}$	**k** $\sqrt{196}$	**l** $\sqrt[3]{3375}$

10 Find the value of each of the following by first evaluating any indices and roots.

a $2^3 \times \sqrt{9}$	**b** $3^3 + \sqrt{36}$	**c** $7^2 \times 10^3$
d $4^3 \div \sqrt{16}$	**e** $2^6 \times 8^2$	**f** $100^2 \times \sqrt{100}$

11 Find the value of each of the following by first evaluating any indices and roots.

a $6^2 \times 3^3$	**b** $18^2 \div 3^2$	**c** $36^2 \div 6^3$
d $4^5 \times 2^1$	**e** $12^2 \times 4^2 \div 3^2$	**f** $\sqrt{81} \times 2^4 \times 5^2$

12 When a square or cube root is multiplied by a number, we can write the number in front of the root without a multiplication sign. For example, $7\sqrt{36} = 7 \times \sqrt{36}$.

Calculate the following by first evaluating the roots.

a $7\sqrt{9}$	**b** $6\sqrt{100}$	**c** $5\sqrt[3]{27}$	**d** $7\sqrt[3]{125}$

13 **a** Find the value of $\sqrt[3]{64}$.

b Find the value of $3\sqrt{64}$.

c Explain the difference between $\sqrt[3]{64}$ and $3\sqrt{64}$.

14 Sally and Jo have the same number of stickers. Sally's stickers are in packs of 15 and Jo's stickers are in packs of 12. What is the smallest number of stickers each of them could have?

15 Taran is a florist. He receives a delivery of roses that can be divided into bouquets of 15 or 24.

a What is the smallest number of roses in the delivery?

b Taran has 24 roses and 40 lilies. If he wants to make identical bouquets with no flowers left over, what is the greatest number of bouquets he can make?

16 A group of students is going on an excursion to Australia Zoo. The students can travel on either buses that seat 30 or buses that seat 45 with all seats being occupied. What is the smallest number of students going on the excursion?

17 Dizzy sold an equal numbers of wontons and dumplings today. Wontons are sold in packets of 8 while dumplings are sold in packets of 12. What is the smallest number of each that Dizzy sold?

18 Arvin has 24 toy trucks and 64 toy cars. He divides them into identical groups so that there are no toys left over. What is the largest number of groups there might be?

19 Jill has grown three types of potatoes in her garden: Pontiac, Sebago and Desiree. Seven Pontiac potatoes weigh the same as four Sebago potatoes and five Sebago potatoes weigh the same as six Desiree potatoes. Put the potatoes in order of weight from lightest to heaviest.

20 How many numbers in the set $\{7^1, 7^2, 7^3, 7^4, \ldots, 7^{2000}\}$ have a last digit of 3?

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

1A **1** Round each number to the nearest hundred.

a 576
b 934
c 7871
d 2839
e 45 621
f 76 618

1A **2** Estimate the result of each calculation by first rounding each number to its leading digit.

a 849 + 381
b 56 – 18
c 65 × 43
d 582 ÷ 16
e 493 × 28
f 825 + 9114
g 8402 ÷ 79
h 658 – 180

1B **3** Calculate each of the following.

a 64 + 98
b 745 + 832
c 6982 + 5176
d 28 + 743 + 6287
e 6287 + 341 + 89
f 3916 + 5328 + 6124

1B **4** Calculate each of the following.

a 63 – 45
b 582 – 96
c 846 – 757
d 5723 – 68
e 8743 – 374
f 7398 – 3572

1B **5** Calculate each of the following.

a 68 – 34 + 287
b 738 + 296 – 457
c 5921 + 4827 – 4277
d 83 422 + 82 113 – 57 329

1C **6** Calculate each of the following.

a 768 × 43
b 846 × 76
c 6214 × 57
d 9262 × 83
e 7248 × 82
f 6524 × 645

1C **7** Use short division to calculate each quotient and remainder.

a 395 ÷ 4
b 646 ÷ 7
c 8328 ÷ 5
d 76 398 ÷ 9
e 68 341 ÷ 11
f 653 789 ÷ 6

1D **8** Find the lowest common multiple (LCM) of each pair of numbers.

a 7 and 6
b 4 and 9
c 5 and 12
d 30 and 45
e 18 and 24
f 36 and 54

1D **9** Find the highest common factor (HCF) of each pair of numbers.

a 24 and 30
b 27 and 72
c 45 and 75
d 120 and 185
e 450 and 600
f 288 and 432

1D **10** Find the value of each of the following by first evaluating any indices and roots.

a $6^2 + 4^3$
b $9^2 \div 3^2$
c $\sqrt{81} \times 4^2$
d $2^6 \div 8$
e $12^2 + 6^2 \div 3^2$
f $\sqrt{49} \times 3^3 + 8^2$

1E Negative integers

Learning intentions

By the end of this topic you will be able to ...

- ✔ compare the value of positive and negative numbers
- ✔ use negative numbers to represent real-life situations.

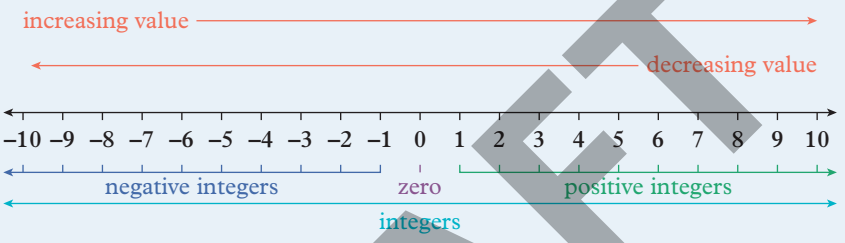
Inter-year links

Support Negative numbers

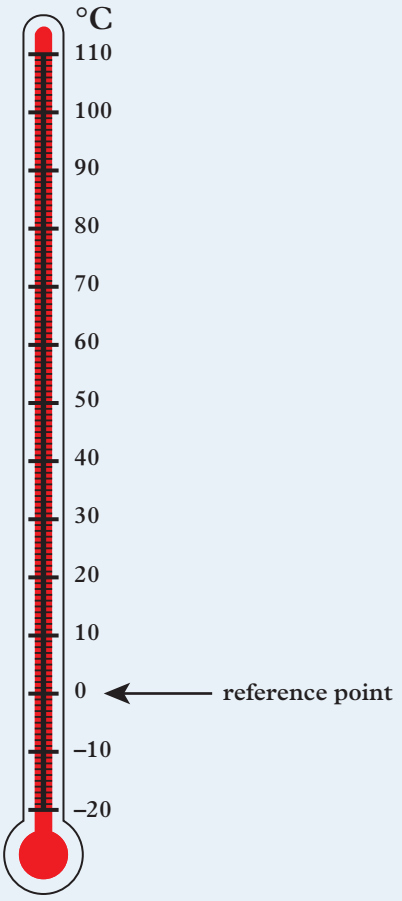
Year 7 5A Negative numbers

Integers

- The set of **integers** includes all positive whole numbers, negative whole numbers and zero.
- **Negative integers** are less than zero in value and are preceded by a negative sign, −, placed directly before the first digit.
- **Positive integers** are greater than zero in value and can be represented by using a positive sign, +. The positive sign can also be left out.

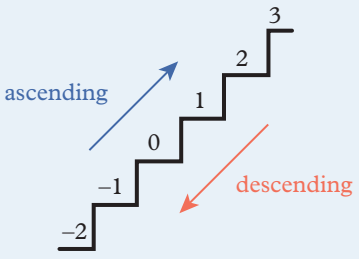


- In the real world, negative numbers are used to represent quantities that lie on a scale that goes below zero. A sign is used to indicate the direction relative to the **reference point** of zero. For example, a negative temperature indicates that the temperature has dropped below the reference point of 0°.



Ordering numbers

- Numbers are in **ascending order** when they are arranged in order of increasing value.
- Numbers are in **descending order** when they are arranged in order of decreasing value.

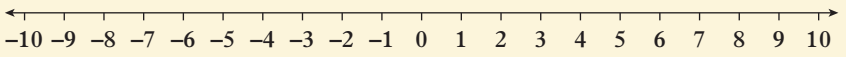


- 'Less than' is shown using the symbol '<'.
- 'Greater than' is shown using the symbol '>'.
- 'Equal to' is shown using the symbol '='.

Example 1E.1 Identifying the greater of two integers

Identify which integer is greater in each pair of numbers. Use the following number line to help you.

a −5 and 2 **b** −4 and −7



THINK

a **1** Locate the two numbers on the number line and compare their positions. 2 is to the right of −5, so 2 is greater than −5.

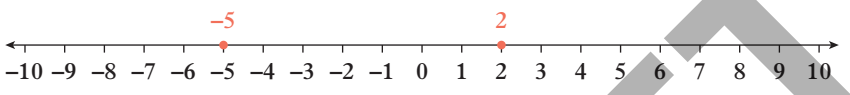
2 Use the 'greater than' symbol, >, in your answer.

b **1** Locate the two numbers on the number line and compare their positions. −4 is to the right of −7, so −4 is greater than −7.

2 Use the 'greater than' symbol, >, in your answer.

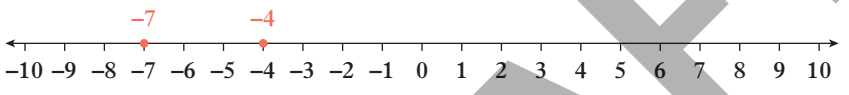
WRITE

a



$2 > -5$

b



$-4 > -7$

Example 1E.2 Representing a situation with an integer

Provide an integer to represent each situation.

a The temperature is 15 degrees above zero.

b A fish swims 4 m below the surface of the water.

c Your bank account is overdrawn by $30.

THINK	WRITE
a **1** The reference point is 0°.	**a** 15
2 Determine the sign of the integer based on the number's position relative to the reference point. '15 degrees above zero' can be indicated using 15.	
b **1** The reference point is the water surface.	**b** −4
2 Determine the sign of the integer based on the number's position relative to the reference point. '4 m below' can be indicated using −4.	
c **1** The reference point is $0.	**c** −30
2 Determine the sign of the integer based on the number's position relative to the reference point. 'Overdrawn by $30' can be indicated using −30.	

Helpful hints

- ✔ The negative equivalent of a positive number is called the additive inverse.
  For example, −4 is the additive inverse of 4.
- ✔ Don't get your 'greater than' and 'less than' symbols mixed up! The 'mouth' of the symbol is always trying to 'eat' the greater number.
  For example, $4 > 3$ and $3 < 4$.

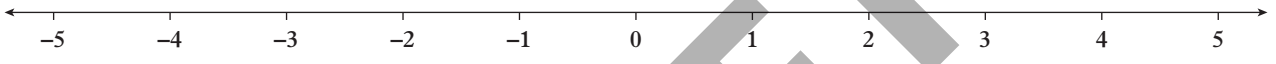
ANS p492

Exercise 1E Negative integers

△ 1–5, 6(a–f), 7(a, c), 8(a, c), 9, 10(a–d), 11, 12 ▢ 4, 5, 6(g–l), 7(b, d), 8(b, d), 9, 10(e–h), 12–15 ⬠ 6(g–l), 7(c, d), 8(c, d), 9, 10(e–h), 13–17

1 For the numbers shown in the following number line, write:

a all the negative integers **b** all the positive integers **c** the integer that is neither positive nor negative.



1E.1 **2** Which integer is greater in each pair of numbers?

a 3 and 9 **b** 0 and 5 **c** −7 and 1
d 5 and −8 **e** 0 and −2 **f** −6 and −4

3 Which integer is smaller in each pair of numbers?

a 5 and 6 **b** 0 and 3 **c** −2 and 7
d 4 and −5 **e** 0 and −1 **f** −8 and −9

4 Which number is greater in each pair of numbers?

a 0 and −38 **b** −42 and 24 **c** 14 and −16
d 125 and −50 **e** −25 and −35 **f** −14 and 0

5 Which number is smaller in each pair of numbers?

a 7 and −70 **b** −21 and −12 **c** 55 and −68
d −40 and −50 **e** −354 and −345 **f** 450 and −450

6 Complete each number statement by writing < (is less than) or > (is greater than) in the space provided. You can draw a number line to help you think.

a −3 ____ 2 **b** −8 ____ −4 **c** 0 ____ −1
d 9 ____ −9 **e** −3 ____ −5 **f** −7 ____ −2
g 4 ____ −6 **h** 5 ____ −12 **i** 6 ____ −4
j −18 ____ −8 **k** −6 ____ −7 **l** −8 ____ 0

7 Write each list of integers in ascending order.

a 14, −20, 10, −7, 8, −11, −12
b −33, 42, −19, −41, 0, 6, 29
c −8, −88, 68, 8, −28, −18, −48
d 73, −82, 3, −140, 104, 145, −126

8 Write each list of numbers in descending order.

a −15, 71, −27, −10, 4, −9

b −1, −2, 5, 10, −11, −5

c 0, 5, −9, −14, 21, −19

d −11, 51, −71, −62, 71, 12, −72

9 List all the integers between −7 and +4 ('between' means −7 and +4 are not included).

1E.2 **10** Provide an integer to represent each situation.

a The lift stops at the third floor below the ground floor.

b You have $2174 in the bank.

c The temperature inside a freezer is 18 degrees below zero.

d The shoreline of the Dead Sea, Jordan, is 408 m below sea level.

e The top of Mt Kilimanjaro, Tanzania, is 5895 m above sea level.

f A skydiver is 500 m above the ground.

g Your bank account is overdrawn by $46.

h A submarine is 40 m below sea level.

11 Polar bears live close to the edge of sea ice in the Arctic, where in winter the temperatures range from around −45°C to 0°C. In summer, the temperatures range from −10°C to 10°C. The ocean temperature is about −2°C.

a Show −45°C, 0°C, −10°C, 10°C and −2°C on a number line.

b Is −10°C higher or lower than −45°C?

c Write the five temperatures in order from lowest to highest.

d A polar bear dips her paw into the water to catch a fish. On a day with an air temperature of −18°C, does the water feel warmer or colder than the air?

12 Automatic teller machines (ATMs) allow you to deposit and withdraw money from your bank account.

a Ashwan checks his bank balance and sees he has $86 in his account. Write this as a positive integer.

b The bank allows Ashwan's account to be overdrawn. This means that he can withdraw more money than he actually has in his account. If he withdraws $100 at an ATM, how much does he owe the bank?

c Write his new bank balance as a negative integer.

13 Jasmine needs glasses for reading as she is long sighted (hypermetropic). The prescription for the lenses in her glasses is +2.50. Alex wears contact lenses because he is short sighted (myopic). His lenses have prescriptions of −3.50 for his left eye and −4.25 for his weaker right eye.

a If Justin has a prescription of +1.75, is he long or short sighted?

b If Elle has a prescription of −4.75, is she long or short sighted?

c How does Elle's prescription compare to Alex's? Who has weaker vision?

d What do you think the reference of zero means in this situation?

14 Bank statements usually show a transaction as a positive number if an amount is added or credited to the account, or as a negative number for withdrawals or debits.

Use the following bank statement to answer these questions.

Transaction	Balance
	+$52
+23	+$75
−35	+$40
−51	−$11
+34	+$23
+13	+$36
−49	−$13

a What is the largest amount:

i credited (added) to the account **ii** debited (subtracted) from the account?

b What is the smallest amount:

i credited to the account **ii** debited from the account?

c What does the final balance indicate about the account?

15 One way to represent integers is with coloured dots. Use one blue dot to represent +1 and one red dot to represent −1.

a How many red dots would you use to represent −3?

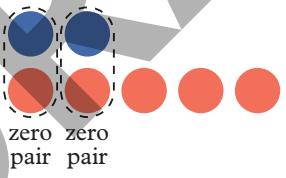
b Use the coloured dots to show each integer listed below. Draw each arrangement.

i 2 **ii** −4 **iii** 6

iv −5 **v** −1 **vi** 8

c One blue dot and one red dot together are called a zero pair. Explain how a zero pair represents 0.

d Explain how this arrangement of dots represents −3.



e What integer does each arrangement represent?

i

ii

iii

16 Some friends are sitting in a circle. Kesha sits opposite Bri and three places to the left of Mo. Dee sits opposite Mo and three places to the left of Kesha. How many chairs are in the circle?

17 Find five consecutive numbers that add to −35.

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Ordering positive and negative numbers

Investigation
Comparing world temperatures

Topic quiz
1E

1F Adding and subtracting integers

Learning intentions

By the end of this topic you will be able to ...

✔ solve problems involving addition and subtraction of positive and negative numbers.

Inter-year links

Year 7 — 5B Adding and subtracting positive numbers

Adding and subtracting positive integers

- When adding a positive integer, move to the right on the number line.

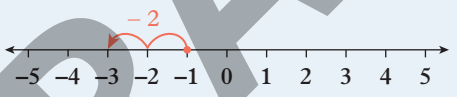
  For example, to calculate $-4 + 5$, start at -4 and move 5 steps to the right.

  $-4 + 5 = 1$

- When subtracting a positive integer, move to the left on the number line.

  For example, to calculate $-1 - 2$, start at -1 and move 2 steps to the left.

  $-1 - 2 = -3$

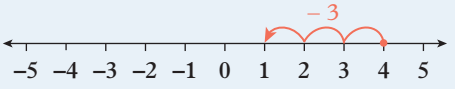


Adding and subtracting negative integers

- When adding a negative integer, move to the left on the number line.

  For example, to calculate $4 + (-3)$, start at 4 and move 3 steps to the left.

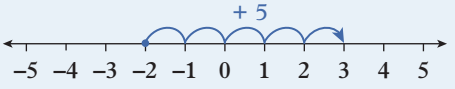
  $4 + (-3) = 4 - 3$
  $= 1$



- When subtracting a negative integer, move to the right on the number line.

  For example, to calculate $-2 - (-5)$, start at -2 and move 5 steps to the right.

  $-2 - (-5) = -2 + 5$
  $= 3$



- Adding a negative number can be simplified to subtracting its value. $+(-) = -$
- Subtracting a negative number can be simplified to adding its value. $-(-) = +$

Example 1F.1 Adding and subtracting positive numbers

Use the number line to calculate:

a $-6 + 8$ **b** $4 - 8$

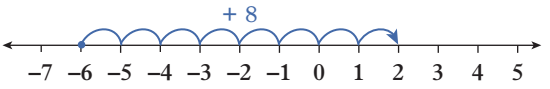
THINK

a 1 Draw a number line.
2 Start at the first number, then add the second number by moving right.

b 1 Draw a number line.
2 Start at the first number, then subtract the second number by moving left.

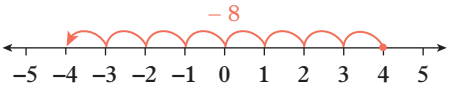
WRITE

a



$-6 + 8 = 2$

b



$4 - 8 = -4$

Example 1F.2 Adding and subtracting negative numbers

Use the number line to calculate:

a $4 + (-7)$ **b** $-2 - (-3)$

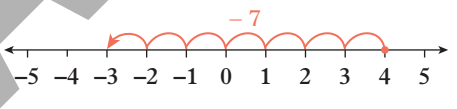
THINK

a 1 Simplify the calculation. Remember that $+(-) = -$.
2 Draw a number line.
3 Start at the first number, then subtract the value by moving left.

b 1 Simplify the calculation. Write the problem in a simpler equivalent form by replacing $-(-)$ with $+$.
2 Draw a number line.
3 Start at the first number, then add the value by moving right.

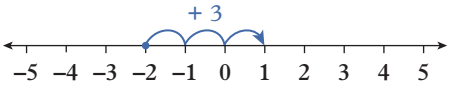
WRITE

a $4 + (-7) = 4 - 7$



$4 + (-7) = -3$

b $-2 - (-3) = -2 + 3$



$-2 - (-3) = 1$

Helpful hints

✔ The negative sign is identical to the minus sign. The function of the symbol depends on its context.

Example	Function of '−'	Read as
-8	Indicates the number is less than zero in value	'negative eight'
$2 - 8$	Indicates the operation of subtraction	'two minus eight'
$-8 + 2$	Indicates the number is less than zero in value	'negative eight plus two'

✔ Don't make assumptions about the sign of the answer – subtraction can produce positive numbers and addition can produce negative numbers!

✔ When simplifying calculations involving negative numbers, only combine the signs located in between the numbers.

$+(-) = -$ $\quad -(-) = +$ $\quad +(+) = +$ $\quad -(+) = -$

ANS p493

Exercise 1F Adding and subtracting integers

▲ 1–3, 4–6(a–e), 7, 8–9(a–e), 10(a, b, d, e), 11(a), 12–14(1st column), 15, 17, 18

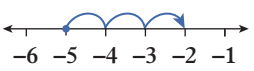
■ 4–6(e–i), 7(a, c, f), 8–10(c–f), 11(b), 12–14(2nd column), 16, 18, 19, 21, 22(a, b)

⬟ 8–9(f–i), 10(f–h), 11(b), 12(c, f, i), 13–14(2nd column), 18, 20–23

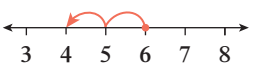
UNDERSTANDING AND FLUENCY

1 Write the addition or subtraction calculation shown in each diagram. Hint: Consider the starting point, the number of steps and the end point.

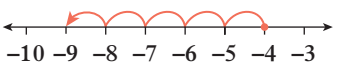
a



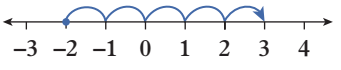
b



c

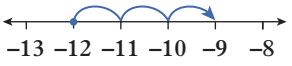


d

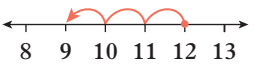


2 Match each diagram (**A**, **B**, **C**) to the appropriate problem (**a**, **b**, **c**).

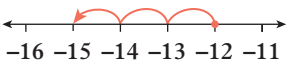
A



B



C



a $-12 - 3$ **b** $-12 + 3$ **c** $12 - 3$

3 Calculate each result for question **2**.

1F.1 **4** Use a number line to calculate:

a $-4 + 7$ **b** $1 + 5$ **c** $8 - 3$

d $-2 - 6$ **e** $-9 + 5$ **f** $3 - 6$

g $-5 - 1$ **h** $7 - 2$ **i** $2 - 6$

5 Draw a number line from −20 to 20 and use it to calculate:

a $15 - 12$ **b** $-17 + 11$ **c** $9 - 13$

d $-4 + 20$ **e** $-2 - 17$ **f** $8 - 18$

g $-14 - 3$ **h** $-16 + 14$ **i** $11 - 24$

j $-19 + 19$ **k** $-10 - 10$ **l** $-18 + 20$

6 Decide whether each result is positive, negative or zero.

a $-3 - 7$ **b** $16 + 14$ **c** $8 - 15$

d $-13 + 13$ **e** $-11 + 18$ **f** $19 - 12$

g $-17 + 4$ **h** $-14 - 10$ **i** $17 - 20$

7 Simplify the following calculations by using + or − to fill the gap.

a $-3 - (+7) = -3$ _____ 7 **b** $1 + (+6) = 1$ _____ 6

c $-4 + (-5) = -4$ _____ 5 **d** $2 - (-4) = 2$ _____ 4

e $5 - (+9) = 5$ _____ 9 **f** $-6 - (-8) = -6$ _____ 8

8 Simplify each problem.

a $-1 - (-3)$ **b** $8 - (+4)$ **c** $-5 + (-2)$

d $6 - (-1)$ **e** $4 + (+3)$ **f** $3 - (+9)$

g $-5 - (-8)$ **h** $-3 + (+1)$ **i** $-7 + (-3)$

1F.2 **9** Use a number line to calculate:

a $-4 - (-7)$ **b** $9 + (-3)$ **c** $2 - (+6)$

d $-5 - (-5)$ **e** $-8 + (+7)$ **f** $6 + (-1)$

g $-3 - (-4)$ **h** $4 - (+9)$ **i** $2 - (-7)$

10 Decide whether the result of each problem will be positive, negative or zero.

a $11 - (-8)$ **b** $-16 - (+4)$ **c** $13 - (+13)$ **d** $15 - (+18)$

e $-10 - (-7)$ **f** $-17 - (+11)$ **g** $8 - (-14)$ **h** $12 - (+19)$

11 Complete each addition table.

a

+	−5	−3	0	1	4
−3				−2	
−2					
−1			−1		
3					
6	1				

b

+	−10	−14	−7	13	19
11					
14					
−10					
−22					
−18					

12 Calculate:

a $-45 + 40$ **b** $-72 - 27$ **c** $56 - 88$

d $-34 + 43$ **e** $130 - 170$ **f** $-213 - 62$

g $-158 + 400$ **h** $286 - 168$ **i** $-505 + 505$

13 Calculate:

a $-2 + 5 + 7$ **b** $3 - 9 + 2$

c $6 - 1 - 8$ **d** $-10 + 4 - 7$

e $-12 + 19 - 7$ **f** $-20 - 11 - 2$

g $33 + 22 - 65$ **h** $70 - 81 - 15$

i $-24 - 18 + 12$ **j** $59 - 93 + 17$

14 Simplify each problem, and then calculate the result.

a $-5 - (+7) + (+6)$ **b** $9 + (-2) - (-8)$

c $-4 - (-3) + (+1)$ **d** $8 + (-6) - (-10)$

e $-22 - (+5) + (+13)$ **f** $-14 - (-8) + (-25)$

g $34 + (-19) - (+7)$ **h** $-9 - (+26) - (-37)$

15 You enter a lift at the 15th floor and travel down 19 floors. Which level do you finish at? Show how you calculated your answer.

16 Sarah enters a lift at the third floor and travels down nine floors and then up five floors. What floor does she finish at? Show the problem you used to get your answer.

17 What is the difference between a temperature of −18°C and a temperature of 33°C? Show the subtraction problem you used to calculate the result.

18 Jess owes her brother $60.

a Write this amount as an integer.

b She pays him $35. Write an addition problem to work out the amount she still owes him.

c How much does Jess still owe her brother?

PROBLEM SOLVING AND REASONING

19 A bungee jumper dives from a platform (position A) 50 m above the water. He dips 2 m under the water (position B) before rebounding to a height 18 m below the platform (position C).

a If the reference is the surface of the water, write positions A, B and C as integers.

b Write an expression to find the distance between:

i A and B

ii B and C

iii A and C.

c Calculate each distance.

d What is the largest distance for part **c**? Explain why that is the largest distance.

20 A deep sea submersible is lowered into the ocean from a ship's deck 15 m above sea level. The unmanned vessel is on a mission to take photos of deep sea animals.

For each part, write an addition or subtraction problem and then calculate the answer.

a A deep sea anglerfish is 900 m below sea level. What vertical distance has the submersible travelled from the deck of the ship to reach this fish?

b A deep sea amphipod is 2500 m below sea level. How much further has the submersible travelled vertically to reach it?

c What vertical distance does the submersible now need to travel to return to the deck of the ship?

d When the ship gets back to the dock, the crane on board the ship lowers the submersible onto the dock. The crane lifts the submersible 2 m above the ship's deck, moves it horizontally at that height and then lowers it 8 m onto the dock. What is the height of the dock above sea level?

CHALLENGE

21 What number should be subtracted from each of these to give the result of 20?

a 65 **b** −10 **c** 1281

d −347 **e** −1000 **f** −3240

22 Find the missing integer to make each number sentence true.

a $-54 + 33 + \_\_\_\_ = 12$ **b** $121 - \_\_\_\_ - 50 = -6$

c $-300 + 225 - \_\_\_\_ = 50$ **d** $\_\_\_\_ - 58 + 73 = -25$

23 At the start of March, Tyra's bank account shows a balance of −$310. At the end of the month, it shows a balance of −$247.

a Calculate the difference between the two account balances. Show the subtraction problem you used to calculate the result.

b Interpret the result to part **a**.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Adding and subtracting with negative numbers

Investigation
Create a subtraction

Topic quiz
1F

1G Multiplying and dividing integers

Learning intentions

By the end of this topic you will be able to ...

- ✔ solve problems involving multiplication and division of negative numbers
- ✔ solve problems involving indices of negative numbers.

Inter-year links

Support	Multiplying whole numbers
Year 7	1D Multiplying whole numbers

Multiplying integers

- Multiplication is a form of repeated addition. For example, 3×4 is the same as $4 + 4 + 4$.
- Multiplication of negative integers can also be thought of as repeated addition. For example, 2×-5 is the same as $(-5) + (-5)$.
- When multiplying by a negative value, we are taking the negative value of the repeated addition. For example, -2×-5 is the same as $-[(-5) + (-5)]$.
- Multiplying two integers can give a result that is positive or negative (or zero), according to these rules.
  - → The product of two positive integers is a positive integer.
  - → The product of a positive integer and a negative integer is a negative integer.
  - → The product of two negative integers is a positive integer.

$$+ \times + = +$$
$$+ \times - = -$$
$$- \times + = -$$
$$- \times - = +$$

Dividing integers

- Division of integers follows the same rules as multiplication of integers.
  - → The quotient of two positive integers is a positive integer.
  - → The quotient of a positive integer and a negative integer is a negative integer.
  - → The quotient of two negative integers is a positive integer.

$$+ \div + = +$$
$$+ \div - = -$$
$$- \div + = -$$
$$- \div - = +$$

Indices of negative numbers

- When applying indices to negative numbers, the sign needs to be taken into account.

index

base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

index form expanded form basic numeral

  - → If the base is negative and the index is an even number, the basic numeral is positive.
  - → If the base is negative and the index is an odd number, the basic numeral is negative.

Example 1G.1 Multiplying integers

Evaluate each product.

a -4×7 **b** $-9 \times (-5)$

THINK	WRITE
a 1 Determine whether the result is positive or negative. The signs are different, so the result is negative.	**a** $4 \times 7 = 28$
2 Calculate the product.	So, $-4 \times 7 = -28$
b 1 Determine whether the result is positive or negative. The signs are the same, so the result is positive.	**b** $9 \times 5 = 45$
2 Calculate the product.	So, $-9 \times (-5) = 45$

Example 1G.2 Dividing integers

Evaluate each quotient.

a $-30 \div (-5)$ **b** $28 \div (-7)$

THINK	WRITE
a 1 Decide whether the result is positive or negative. The signs are the same, so the result is positive.	**a** $30 \div 5 = 6$
2 Calculate the quotient.	So, $-30 \div (-5) = 6$
b 1 Decide whether the result is positive or negative. The signs are different, so the result is negative.	**b** $28 \div 7 = 4$
2 Calculate the quotient.	So, $28 \div (-7) = -4$

Example 1G.3 Multiplying three integers

Evaluate the product $-3 \times (-5) \times (-2)$.

THINK	WRITE
1 Work from left to right. Multiply the first two numbers and determine the sign. As the signs are the same, the result is positive.	$-3 \times (-5) = 15$
2 Multiply the result by the third number and determine the sign. As the signs are different, the result is negative.	$15 \times (-2) = -30$ $-3 \times (-5) \times (-2) = -30$

Example 1G.4 Calculating the value of a number in index form

Write each of these in expanded form and calculate their values.

a $(-3)^2$ **b** $(-2)^5$

THINK

a 1 Write the calculation as a repeated multiplication.

2 Determine whether the result will be positive or negative. A negative multiplied by a negative is a positive.

3 Calculate the product.

b 1 Write the calculation as a repeated multiplication.

2 Continue to simplify the repeated multiplication. A negative multiplied by a negative is a positive. A positive multiplied by a negative is a negative.

3 Calculate the product.

WRITE

a $(-3)^2 = (-3) \times (-3)$
$= 9$

b $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$
$= 4 \times (-2) \times (-2) \times (-2)$
$= (-8) \times (-2) \times (-2)$
$= 16 \times (-2)$
$= -32$

Helpful hints

✔ Be careful when determining the sign of a result when multiplying or dividing numbers.
If the signs are the same, then the result is positive.
If the signs are different, then the result is negative.

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Exercise 1G Multiplying and dividing integers

1, 2–3(1st, 2nd columns), 4–6, 7(a), 8–14, 15(a–c), 16(a, b), 17, 19, 20(a, b, e), 22(a–c)

4–6, 7(b), 8, 9–10(d–f), 13(e–h), 14(c, d), 15(d–f), 16(c, d), 18, 20(c, d, f), 22, 23(a, b)

5(i–l), 7(b), 8, 9(g–i), 10(b, d, e), 14(c, d), 15(g–i), 20, 21, 23–26

1G.1 **1** Express these repeated additions as multiplications.

a $9 + 9 + 9 + 9 + 9$ **b** $(-7) + (-7) + (-7) + (-7) + (-7) + (-7)$

c $-(6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6)$ **d** $-[(-3) + (-3) + (-3) + (-3)]$

1G.1 **2** Evaluate each product.

a $3 \times (-4)$ **b** 7×5 **c** -6×2 **d** $-3 \times (-9)$

e -1×8 **f** $4 \times (-6)$ **g** $-8 \times (-9)$ **h** 2×10

i -4×11 **j** $7 \times (-1)$ **k** $-2 \times (-4)$ **l** 9×5

1G.2 **3** Evaluate each quotient.

a $15 \div (-3)$ **b** $-32 \div (-8)$ **c** $24 \div 4$ **d** $-63 \div 9$

e $-17 \div (-1)$ **f** $48 \div (-6)$ **g** $-56 \div 8$ **h** $-81 \div (-9)$

i $42 \div (-7)$ **j** $-12 \div 1$ **k** $-20 \div (-2)$ **l** $72 \div (-8)$

4 Evaluate each quotient.

a $\frac{-48}{6}$ **b** $\frac{60}{-5}$ **c** $\frac{-51}{-3}$ **d** $\frac{150}{10}$ **e** $\frac{-200}{-25}$ **f** $\frac{57}{-19}$

5 Calculate each of the following.

a $5 \times (-7)$ **b** $-9 \div 3$ **c** $-28 \div (-4)$ **d** -10×6

e $36 \div (-9)$ **f** $-15 \times (-4)$ **g** $-100 \div 20$ **h** $18 \times (-2)$

i $-3 \times (-30)$ **j** $25 \div (-25)$ **k** -1×38 **l** $-45 \div (-1)$

6 Complete each statement.

a $-15 \times$ _____ $= 45$ **b** $56 \div$ _____ $= -8$ **c** _____ $\times (-4) = 36$

d _____ $\div (-25) = -4$ **e** $-140 \div$ _____ $= 2$ **f** _____ $\times 16 = -48$

7 Complete each multiplication table.

a

×	−2	−1	0	1	2
−4	8			−4	
−2					
0			0		
2					
4		−4			
6					

b

×		−20	−10		20
7				70	
			−50		
0					
−2	50				
		80			
					−120

8 A short way of writing $-1 \times (-2)$ is $-(-2)$. Write each problem as the product of two numbers and then calculate the result.

a $-(-7)$ **b** $-(+12)$ **c** $-(-88)$

d $-(+25)$ **e** $-(8 - 5)$ **f** $-(-3 + 2)$

1G.3 **9** Evaluate each product.

a $-4 \times 3 \times (-5)$ **b** $3 \times (-2) \times 7$ **c** $6 \times (-1) \times 9$

d $8 \times 2 \times (-3)$ **e** $-5 \times (-2) \times (-13)$ **f** $-7 \times 3 \times (-4)$

g $4 \times 5 \times (-8)$ **h** $-11 \times 5 \times (-2)$ **i** $9 \times (-4) \times 5$

10 Evaluate each statement.

a $-2 \times 2 \times (-3) \times 3$ **b** $20 \div (-2) \div (-2) \div (-1)$ **c** $-4 \times (-1) \times 3 \times (-2)$

d $-18 \div (-2) \times 4 \div (-6)$ **e** $9 \times (-2) \div 3 \times (-5)$ **f** $-7 \times (-4) \div 2 \times (-3)$

11 Which of these five options is the expanded form of $(-4)^5$?

A $4 \times 4 \times 4 \times 4 \times 4$ **B** $5 \times 5 \times 5 \times 5$ **C** $-4 \times (-4) \times 4 \times 4 \times 4$

D $-4 \times (-4) \times (-4) \times (-4)$ **E** $-4 \times (-4) \times (-4) \times (-4) \times (-4)$

12 Without calculating each result, predict whether the basic numeral is positive or negative.

a $(-2)^9$ **b** $(-15)^6$ **c** $(+6)^7$ **d** $(-100)^4$

1G.4 **13** Write each of these in expanded form and calculate their values.

a $(-9)^2$ **b** 8^2 **c** $(-6)^3$ **d** 7^3

e $(-5)^4$ **f** 1^4 **g** 4^5 **h** $(-10)^5$

14 By collecting like terms, write each of these multiplications in index form.

a $-5 \times (-5) \times (-5) \times (-9) \times (-9) \times (-9) \times (-9)$

b $-4 \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) \times 3 \times 3 \times 3$

c $7 \times 7 \times (-6) \times (-6) \times (-6) \times (-6) \times (-6) \times (-6)$

d $-8 \times (-8) \times (-8) \times (-8) \times (-10) \times (-10)$

15 Write each of these in expanded form and calculate their values.

a $(-2)^3 \times (-4)^2$ **b** $(-5)^2 \times 3^4$ **c** $(-10)^3 \times (-2)^5$

d $(-3)^5 \times (-1)^2$ **e** $(-6)^2 \times (-2)^3$ **f** $(-1)^4 \times 3^2$

g $(-7)^2 \times (-1)^5$ **h** $(-3)^3 \times (-2)^2$ **i** $(-1)^3 \times (-2)^5$

16 Complete each set of problems. Describe the pattern you see.

a

$2 \times 5 =$ _____
$2 \times 4 =$ _____
$2 \times 3 =$ _____
$2 \times 2 =$ _____
$2 \times 1 =$ _____
$2 \times 0 =$ _____
$2 \times (-1) =$ _____
$2 \times (-2) =$ _____
$2 \times (-3) =$ _____
$2 \times (-4) =$ _____
$2 \times (-5) =$ _____

b

$-3 \times 5 =$ _____
$-3 \times 4 =$ _____
$-3 \times 3 =$ _____
$-3 \times 2 =$ _____
$-3 \times 1 =$ _____
$-3 \times 0 =$ _____
$-3 \times (-1) =$ _____
$-3 \times (-2) =$ _____
$-3 \times (-3) =$ _____
$-3 \times (-4) =$ _____
$-3 \times (-5) =$ _____

c

$-4 \times (-5) =$ _____
$-4 \times (-4) =$ _____
$-4 \times (-3) =$ _____
$-4 \times (-2) =$ _____
$-4 \times (-1) =$ _____
$-4 \times 0 =$ _____
$-4 \times 1 =$ _____
$-4 \times 2 =$ _____
$-4 \times 3 =$ _____
$-4 \times 4 =$ _____
$-4 \times 5 =$ _____

d

$5 \times (-5) =$ _____
$5 \times (-4) =$ _____
$5 \times (-3) =$ _____
$5 \times (-2) =$ _____
$5 \times (-1) =$ _____
$5 \times 0 =$ _____
$5 \times 1 =$ _____
$5 \times 2 =$ _____
$5 \times 3 =$ _____
$5 \times 4 =$ _____
$5 \times 5 =$ _____

17 Dividing is the inverse operation to multiplying. Complete each statement.

a $2 \times 3 =$ _____ so _____ $\div 2 = 3$ or _____ $\div 3 = 2$

b $2 \times (-3) =$ _____ so _____ $\div 2 = -3$ or _____ $\div (-3) = 2$

c $-2 \times 3 =$ _____ so _____ $\div (-2) = 3$ or _____ $\div 3 = -2$

d $-2 \times (-3) =$ _____ so _____ $\div (-2) = -3$ or _____ $\div (-3) = -2$

18 a Calculate each of these by first writing them in expanded form.

i $(-1)^1$ **ii** $(-1)^2$ **iii** $(-1)^3$

iv $(-1)^4$ **v** $(-1)^5$ **vi** $(-1)^6$

b Which of these problems give a positive result?

c Which of these problems give a negative result?

d Explain your observations.

e Predict the result for:

i $(-1)^{13}$ **ii** $(-1)^{32}$ **iii** $(-1)^{100}$

iv $(-1)^{203}$ **v** $(-1)^{188}$ **vi** $(-1)^{555}$

19 a Using a calculator, determine:

i 8×0 **ii** -15×0 **iii** 0×65 **iv** $0 \times (-72)$

b What do you notice when you multiply any integer by zero?

c Using a calculator, determine:

i $0 \div (-9)$ **ii** $0 \div 26$ **iii** $0 \div (-11)$ **iv** $0 \div 39$

d What do you notice when you divide zero by any integer?

20 Find two integers that have:

a a sum of -9 and a product of 18 **b** a sum of 4 and a product of -45

c a sum of -5 and a product of -66 **d** a difference of 2 and a product of 24

e a difference of 8 and a product of 9 **f** a difference of 18 and a product of -32.

21 The product of 4 integers is 50. What are the possible sums of these 4 integers if they are:

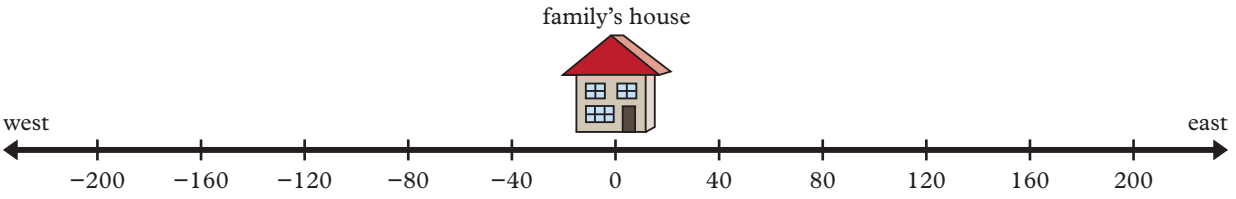
a all positive **b** all negative **c** either positive or negative?

Hint: First find all the factors of 50.

PROBLEM SOLVING AND REASONING

22 Astro the dog has escaped from his yard. His owners decide to search along the road which runs east–west outside the house. This number line can be used to represent the road outside the family's house, with the house at zero.

a Tyler and Rhys walk 120 m west from the house and call Astro's name. What is their position on the number line?



b Sophie and Imogen walk 80 m east from the house and call Astro's name. What is their position on the number line?

c What is the distance between the pairs of searchers?

d Imogen now walks a further 30 m east. What is her new position on the number line?

e Rhys now walks 160 m east. What is his new position on the number line?

f What is the distance between:

i Sophie and Rhys **ii** Tyler and Imogen?

g Natalie starts at the house, then walks twice as far as Sophie but in the same direction as Tyler. What is her position on the number line?

h Hayden starts at the house, then walks one-third of the distance walked by Tyler and in the same direction. What is his position on the number line?

i A neighbour has found Astro in her yard, 60 m west of the family's house. Who is closest to Astro? How far and in which direction does he or she need to walk to collect the dog?

j If each member of the family now walks home, calculate the distance each person walked from the start of the search.

23 Squaring two different numbers can produce the same result. For example, $3^2 = 9$ and $(-3)^2 = 9$.

a Which two numbers, when squared (raised to the power of 2), give 25? Why are there two numbers?

b Is there a number that, when squared, gives −25? Explain.

c How many different numbers can be cubed (raised to the power of 3) to give 8?

d How many different numbers can be cubed (raised to the power of 3) to give −8?

e Predict how many numbers can be raised to the power of 4 to give the same positive result. Hint: Try to raise a number to the power of 4 to give 16. Repeat for a negative result.

f Similarly, predict what would happen for indices of 5, 6 and so on. Provide examples to support your answer.

24 What number(s), when squared, give(s) each result?

a 49 **b** 81 **c** 1 **d** 4 **e** 64 **f** 100

CHALLENGE

25 What number(s), when cubed, give(s) each result?

a 27 **b** −125 **c** 64 **d** −1 **e** −64 **f** −1000

26 Consider $(-3)^2$, 3^2 and -3^2.

a Which of these produce the same result?

b Explain how $(-3)^2$ is different from -3^2.

c Decide whether each pair produces the same result.

i $(-4)^3$ and -4^3 **ii** $(-2)^4$ and -2^4 **iii** $(-1)^6$ and -1^6 **iv** $(-10)^5$ and -10^5

d Describe the general pattern you have seen.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Multiplying and dividing negative numbers
- Interactive skillsheet: Indices of negative numbers
- Investigation: Multiplication triangulation
- Topic quiz: 1G

1H Order of operations

Learning intentions

By the end of this topic you will be able to ...

- ✔ order operations in calculations involving two or more operations
- ✔ evaluate expressions including integers using the order of operations.

Inter-year links

Support — Order of operations

Year 7 — 1H Order of operations

Order of operations

- **'BIDMAS'** can be used to help remember the **order of operations**.

B	**Brackets**	Operations inside brackets are always performed first. For example, $2^2 \times (4 - 6) - 8 \div (-2) = 2^2 \times (-2) - 8 \div (-2)$.
I	**Indices (and roots)**	Next, evaluate indices and roots. For example, $2^2 \times (-2) - 8 \div (-2) = 4 \times (-2) - 8 \div (-2)$.
D **M**	**Division** **Multiplication**	Then working from **left to right**, perform any multiplication or division before adding or subtracting. For example, $4 \times (-2) - 8 \div (-2) = -8 - (-4)$.
A **S**	**Addition** **Subtraction**	Finally, working from **left to right**, perform any addition and subtraction. For example, $-8 - (-4) = -8 + 4 = -4$.

- Operations at the same ranking are performed in order from left to right.

  For example, $5 + 8 - 2 = 13 - 2$
  $= 11$

  and $-20 \times \frac{3}{10} = -\frac{60}{10}$
  $= -6$.

- Where there is more than one set of brackets in the problem, perform the operations inside the innermost brackets first.

  For example, $[2 \times (5 - 3)] - 7 = [2 \times (5 - 3)] - 7$
  $= (2 \times 2) - 7$
  $= 4 - 7$
  $= -3$.

Addition (+)	Subtraction (−)	Multiplication (×)	Division (÷)
• Sum	• Difference	• Product	• Quotient
• Plus	• Take away	• Times	• Divide
• More than	• Less than	• Multiply	• Shared
• Increase	• Fewer	• Groups of	• Distributed
• Total	• Reduce		
• Together	• Minus		

Example 1H.1 Order of operations

Evaluate each of the following.

a $-3 + 4 \times (-2)$ **b** $7 - (-15) \div 5$ **c** $(-2)^3 \times 3 + 5$

THINK

a **1** Perform the multiplication and determine the sign of the result.

2 Simplify the signs and perform the subtraction.

b **1** Perform the division and determine the sign of the result.

2 Simplify the signs and perform the addition.

c **1** Apply the index and determine the sign of the result.

2 Perform the multiplication and determine the sign of the result.

3 Perform the addition.

WRITE

a $-3 + 4 \times (-2) = -3 + (-8)$

$= -3 - 8$

$= -11$

b $7 - (-15) \div 5 = 7 - (-3)$

$= 7 + 3$

$= 10$

c $(-2)^3 \times 3 + 5 = -8 \times 3 + 5$

$= -24 + 5$

$= -19$

Example 1H.2 Order of operations with brackets

Evaluate each of the following.

a $-18 \div (4 - 10) + 2 \times 7$ **b** $6 - [-12 \div (9 - 5)]^2 - 2$

THINK

a **1** Perform any operations inside brackets first.

2 Working from left to right, perform division and multiplication before addition.

3 Perform the addition.

b **1** There are two sets of brackets. Perform the operation in the innermost set of brackets first.

2 Perform the operation in the remaining set of brackets.

3 Apply the index before addition and subtraction.

4 Working from left to right, perform the subtraction.

WRITE

a $-18 \div (4 - 10) + 2 \times 7 = -18 \div -6 + 2 \times 7$

$= 3 + 2 \times 7$

$= 3 + 14$

$= 17$

b $6 - [-12 \div (9 - 5)]^2 - 2 = 6 - [-12 \div 4]^2 - 2$

$= 6 - [-3]^2 - 2$

$= 6 - 9 - 2$

$= -3 - 2$

$= -5$

Helpful hints

- ✔ Remember to use BIDMAS rather than working from left to right.
- ✔ It is helpful to determine the sign at each step of a calculation before progressing.
- ✔ Calculating the value of a number in index form uses the following rules:
  - If the base is negative and the index is an even number, the result is positive.
  - If the base is negative and the index is an odd number, the result is negative.
- ✔ For multiplying and dividing integers, recall the simplification rules for calculations involving two numbers.
  - If the signs are the same, then the result is positive.
  - If the signs are different, then the result is negative.

$+(-)=-$ $\quad -(-)=+$ $\quad +(+)=+$ $\quad -(+)=-$

- ✔ For adding and subtracting integers, recall the simplification rules. You can also use the number line to help you.

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Exercise 1H Order of operations

△ 1–3, 4(1st column), 5–10, 12, 13(a, b) ■ 3, 4(2nd column), 5–9, 11, 13, 16 ⬠ 4(2nd column), 5–8, 13, 14, 16–18

1 For each calculation, list the operations in the correct order in which they should be completed.

a $-28 - 3 \times 7 + 11$ **b** $50 + (34 - 19) \div (-5)$ **c** $12 + 3 \times \sqrt{4} - (-41)$

d $-20 - 2 \times 6 + 3^2$ **e** $8 + [9 - (-3 + 2)]$ **f** $24 \div 6 + [5 + (-1)]^2$

1H.1 **2** Evaluate each of the following.

a $16 + 28 \div 4$ **b** $60 - 2 \times 10$ **c** $27 \div 3 + 6 \times 11$

d $22 + 30 \times 2^3 \div 15$ **e** $10 \times 8 - 7 \times \sqrt{9}$ **f** $6^2 - 8 \times 3 + 35$

3 Evaluate each of the following.

a $7 - 2 \times 8$ **b** $-1 + 16 \div 16$ **c** $28 - 3 \times 11$

d $-6 + 5 \times \sqrt[3]{27} - 2$ **e** $-5 + [-9 + (-6 + 4)]$ **f** $42 \div (-7) + [5 \times (-2)^2]$

1H.2 **4** Evaluate each of the following.

a $72 \div (7 + 2) - 2 \times 3$ **b** $(27 - 14) \times (19 + 11)$

c $11 - (4 \times 2 - 19)$ **d** $(-3 \times 7 + 1) \div (8 - 12)$

e $9^2 + 7 \times (34 - 28)$ **f** $10 \times 8 - 4 \times (3^2 + \sqrt[3]{8})$

g $60 \div [6 + (3 \times 5 - 1)] + 48$ **h** $-12 + [-48 \div (28 - 26)]^2 - 9$

5 Evaluate each of the following.

a $(-3)^2 \times (-14 + 10) \div (-6)$ **b** $(-19 + 16)^2 + (-23 + 17)^3$ **c** $(-2)^3 \times (-4) \div (-8)$

d $(-2)^2 \times (-2^2)$ **e** $(120 \div 6) - (-10) \div \sqrt{4}$ **f** $[-3 + (150 \div 50)] \times (-7)^2 \div 5^2$

6 Write an equivalent sum to the following using a single sign. Do not calculate the answers.

a $66 + (-7)$ **b** $415 - (-322)$ **c** $365 - (+109)$

d $1750 - (-3487)$ **e** $4376 + (-237)$ **f** $6548 - (+2399)$

7 Identify the mistakes that have been made in the following calculations.

a $(-10 + 18)^2 = -64$ **b** $-15 + 5 \times 5 = -50$ **c** $-18 \div 3 + \sqrt{36} = -2$

d $(4 - 20) \div (-4) = -4$ **e** $(-4)^3 \div 8 = 8$ **f** $-5 \times 6 + 90 \div 10 = 39$

8 Find the correct answers to the problems in question **7**.

9 In which of the following do the brackets not change the value of $-2^2 + 5 \times 3^2 - 2 \times \sqrt[3]{8} - 7$?

a $(-2)^2 + 5 \times 3^2 - 2 \times \sqrt[3]{8} - 7$ **b** $(-2^2 + 5) \times 3^2 - 2 \times \sqrt[3]{8} - 7$ **c** $-2^2 + (5 \times 3^2) - 2 \times \sqrt[3]{8} - 7$

d $-2^2 + 5 \times (3^2 - 2) \times \sqrt[3]{8} - 7$ **e** $-2^2 + 5 \times 3^2 - 2 \times (\sqrt[3]{8} - 7)$

10 You have $195 to buy three shirts priced at $58 each.

a Write a calculation to show how to work out how much money you will have left over.

b Estimate how much money is left over by rounding each number to its leading digit then performing the calculation.

c Perform the calculation in part **a** and compare your estimate to the exact amount of money you have left over.

11 Using the images shown, calculate the cost of buying lunch for the class: 14 salad rolls, 11 samosa packs, 9 flavoured milks, 16 orange juices and 25 apples. Show the numbers and operations you used in the calculation.

$7 $6 for three $4 $5 $1

12 When standing with outstretched arms above her head, the height of an Olympic diver is 3 m. She dives from the 10 m platform and touches the bottom of the pool with her outstretched fingers. The depth of the pool is 6 m.

a If the pool surface is the reference point for zero, write the maximum height of the diver above the pool surface as a directed number.

b Write the depth of the pool as a directed number.

c Write a subtraction problem to calculate the vertical distance covered by the diver. Hint: Find the difference between the two positions.

d What vertical distance has the diver covered?

13 The daily maximum and minimum temperatures at Thredbo were recorded over a week.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Maximum temperature (°C)	8	4	−3	−2	−1	0	1
Minimum temperature (°C)	2	−3	−5	−6	−6	−3	0

a Calculate the difference between the minimum and maximum temperatures for each day.

b Which day had the biggest range of temperatures?

c Calculate the average (mean) of:

i the maximum temperatures

ii the minimum temperatures.

d What is the difference between the average minimum and average maximum temperatures?

14 This incomplete statement shows some deposits and withdrawals made to Martin's bank account.

Date	Reference	Transaction	Balance
30 April	–	–	+$289
2 May	Tony's Newsagent	+$132	
15 May	Movie Place	−$17	
21 May	Bicycle City	−$500	
24 May	Tony's Newsagent	+$230	
26 May	DD Online Music Store	−$38	
27 May	Sports and Stuff Pty Ltd		+$69

a Is a transaction of +$132 a deposit or a withdrawal?

b Is a transaction of −$17 a deposit or a withdrawal?

c What does it mean if the balance in Martin's account is:

i positive **ii** negative?

d Find the account balance after the transaction made on 21 May.

e Find the balance after the transaction made on 24 May.

f What transaction is made on 27 May so that the account balance is +$69?

g Was Martin's account overdrawn at any stage? What penalty do banks have for an account that is overdrawn?

15 If the sum of two numbers is zero, is the product of the numbers positive or negative? Explain.

16 Find three integers whose sum is −5 and product is 120.

17 Evaluate the following.

a $6 + \cfrac{52}{6 + \cfrac{63}{3 + \cfrac{48}{3 + \cfrac{55}{3 + \cfrac{96}{8 + \cfrac{40}{8 + \cfrac{4}{1 + \cfrac{5}{1 + \frac{8}{2}}}}}}}}}$

b $4 - \cfrac{-64}{10 - \cfrac{52}{15 - \cfrac{132}{-3 - \cfrac{-54}{-1 - \cfrac{50}{3 - \cfrac{7}{3 - \cfrac{36}{4 - \cfrac{10}{1 - \frac{24}{8}}}}}}}}}$

18 The 'Four Fours' problem uses four of the digit '4' and a combination of operations and brackets to write calculations that are equal to each integer.

For example, 4 + 4 + 4 + 4 = 16.

a Write a series of calculations, using four '4's and any combination of the operations +, −, ×, ÷ and brackets, that are equal to each of the integers 0 to 20. Are all of these integers possible using these operations?

b If we include the square root operation, $\sqrt{4}$, can you write a calculation for each of the integers 1 to 20? If you couldn't find all the numbers in part **a**, see if you can find them using the square root operation.

c The factorial, $n!$, is an operation that multiplies each of the integers from a number, n, down to 1.

For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

Using the factorial with the above operations, which numbers are now possible?

d Explain how you could use your results above to write calculations that are equal to each of the integers from −20 to −1.

PROBLEM SOLVING AND REASONING

CHALLENGE

Chapter summary

Rounding and estimating

$12\,600 \approx 13\,000$ (Round up: 6 → 7)

$12\,300 \approx 12\,000$ (Round down: 3 → 2)

Leading digit

$123\,456 = 123\,456 \approx 100\,000$

Adding and subtracting

$$\begin{array}{r} {\scriptstyle 1} \\ 229 \\ +\ 342 \\ \hline 571 \end{array}$$

Ones column: 9 + 2 = 11, carry the 1
Tens column: 2 + 4 + (1) = 7
Hundreds column: 2 + 3 = 5
← answer

$$\begin{array}{r} {\scriptstyle 1\ 11} \\ 321 \\ -\ 204 \\ \hline 117 \end{array}$$

Ones column: Take 10 from the tens column 11 – 4 = 7
Tens column: 1 – 0 = 1
Hundreds column: 3 – 2 = 1
← answer

The distributive law

$5 \times 32 = 5 \times (30 + 2)$
$= 5 \times 30 + 5 \times 2$
$= 150 + 10$
$= 160$

Multiplication algorithm

$$\begin{array}{rr} & {\scriptstyle 2}\,34 \\ \times & 26 \\ \hline & 204 \quad (34 \times 6) \\ + & 680 \quad (34 \times 20) \\ \hline & 884 \end{array}$$

Multiply by ones →
Multiply by tens →
Add the results →

Short division

quotient: 145 remainder 1
$6\overline{)8^{2}7^{3}1}$
divisor, dividend, remainder

Lowest common multiple (LCM)

Multiples of 4: 4, 8, (12), 16, …
Multiples of 6: 6, (12), 18, 24, …
LCM: 12

Highest common factor (HCF)

Factors of 12: 1, 2, 3, 4, (6), 12
Factors of 18: 1, 2, 3, (6), 9, 18
HCF: 6

Squares and cubes

$1^2 = 1 \times 1 = 1$	$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$	$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$	$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$	$\sqrt[3]{27} = 3$

Integers

increasing value →
← decreasing value

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

negative integers | zero | positive numbers
integers

Adding and subtracting integers

$+\,(-) = -$ $\quad -\,(+) = -$
$-\,(-) = +$ $\quad +\,(+) = +$

Powers of integers

index
base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$
index form | expanded form | basic numeral

Order of operations

B	Brackets	$5 \times (10 - 2) = 5 \times 8 = 40$
I	Indices (and roots)	$3^3 - 4 = 27 - 4 = 23$
D M	Division Multiplication	$5 \times 3 - 4 + 8 \div 4 = 15 - 4 + 2 = 13$
A S	Addition Subtraction	$15 - 4 + 2 = 13$

Multiplying and dividing integers

$+ \times + = +$ $\quad + \div + = +$
$+ \times - = -$ $\quad + \div - = -$
$- \times + = -$ $\quad - \div + = -$
$- \times - = +$ $\quad - \div - = +$

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Mathematical literacy review

The following key terms are used in this chapter:

- addition algorithm
- ascending order
- associative law
- base
- BIDMAS
- by-parts method
- common factor
- commutative law
- compensation method
- cube
- cube root
- descending order
- distributive law
- dividend
- divisor
- estimate
- expanded form
- exponent
- factors
- highest common factor
- index
- integers
- jump method
- leading digit
- lowest common multiple
- multiples
- multiplication algorithm
- negative integers
- order of operations
- place value
- positive integers
- product
- quotient
- reference point
- remainder
- round down
- round up
- rounding
- short division
- square
- square root
- subtraction algorithm

1 Which mental strategy for adding or subtracting numbers involves rounding one number to make the calculations easier, and then adding or subtracting the amount by which the number was rounded up or down?

2 At which stage of BIDMAS are roots evaluated?

3 When performing a division, what is the quotient?

4 Identify the key terms being referenced in each of these definitions.

a the set of positive whole numbers, negative whole numbers and zero

b an approximate value which is close to the actual value

c the result of a multiplication

5 Using an example, provide a definition in your own words for the following key terms.

a multiples **b** square root **c** ascending order

6 Complete the following sentences using words from the key term list.

a The _____ states that the order in which numbers are added does not change the result.

b The _____ of a whole number are the whole numbers which divide exactly into that number.

Multiple choice

1A **1** By first rounding each number to its leading digit, what is an estimate for 526 × 68?

A 36 000 **B** 35 000 **C** 33 664 **D** 31 200 **E** 30 000

1A **2** Which of the following numbers would be 700 when rounded to the nearest hundred?

A 789 **B** 750 **C** 634 **D** 651 **E** 792

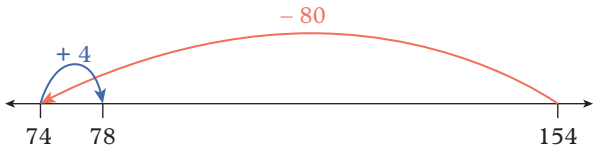
1B **3** What is 45 783 added to 67 398?

A 103 181 **B** 113 081 **C** 113 171 **D** 112 181 **E** 113 181

1B **4** What is 3765 subtracted from 9632?

A 5877 **B** 5967 **C** 6867 **D** 5867 **E** 5977

1B **5** Which calculation is being displayed in this diagram of the compensation method?



A 154 – 76 = 78 **B** 154 – 84 = 78 **C** 154 – 84 = 74
D 74 + 78 = 154 **E** 78 + 84 = 154

1C **6** What is 144 multiplied by 212?
A 356 **B** 30 536 **C** 30 000 **D** 30 528 **E** 28 800

1C **7** What is 672 divided by 12?
A 58 **B** 57 **C** 56 **D** 55 **E** 54

1C **8** What are the quotient and remainder when 1175 is divided by 7?
A quotient = 166, remainder = 1 **B** quotient = 167, remainder = 1
C quotient = 168, remainder = 1 **D** quotient = 166, remainder = 6
E quotient = 167, remainder = 6

1D **9** Which of the following is the HCF of 36 and 90?
A 5 **B** 9 **C** 36 **D** 18 **E** 3

1D **10** Which of the following represents the square root of 36?
A $\sqrt{36}$ **B** $\sqrt{6}$ **C** 36^2 **D** 6^2 **E** 216

1E **11** Which of the following numbers is smaller than –25?
A –18 **B** 2 **C** 25 **D** 0 **E** –30

1E **12** Which list of integers is written in descending order?
A 14, 11, 6, 0, –3, –5, –22 **B** 14, 11, 6, 0, –22, –5, –3
C –22, 14, 11, 6, –5, –3, 0 **D** –22, –5, –3, 0, 6, 11, 14
E 0, 6, 11, 14, –3, –5, –22

1F **13** What integer when added to 12 gives –7?
A –17 **B** –5 **C** 5 **D** 19 **E** –19

1F **14** What is –24 – 20?
A –4 **B** 4 **C** –44 **D** 44 **E** –48

1F **15** What does –35 – (–47) simplify to?
A –35 – 47 **B** 35 – 47 **C** 35 + 47 **D** –35 + 47 **E** –47 + 35

1G **16** What is 8 ÷ (–2)?
A –16 **B** –4 **C** 4 **D** 16 **E** 6

1G **17** What pair of integers gives a sum of –5 and a product of –36?
A –4 and 9 **B** –12 and 3 **C** 4 and 9 **D** –9 and –4 **E** –9 and 4

1G **18** Which gives the largest result?
A $(-2)^6$ **B** 6^2 **C** $(-1)^{10}$ **D** $(-4)^3$ **E** $-(8)^2$

1H **19** Which of these problems does *not* equal 5?
A –4 – 11 + 16 – 2 + 6 **B** 8 – 13 – 1 + 17 – 6 **C** –7 + 5 – 9 + 22 – 6
D 5 – 12 + 3 – 7 + 6 **E** 8 – 9 + 20 – 18 + 4

1H **20** What operation is performed first in $6 \times (12 - 5) + 8^2 \div 4$?
A 12 – 5 **B** 8^2 **C** 6 × 12 **D** 64 ÷ 4 **E** 8 ÷ 4

Short answer

1A **1** Round each number to its leading digit.

a 236 **b** 67 145 **c** 3890 **d** 149 046

1A **2** Estimate the result by first rounding each number to its leading digit.

a 12 345 + 3648 **b** 94 501 − 32 566 **c** 394 × 338 **d** 18 654 ÷ 425

1B **3** Three houses in one street were all sold on the same day. The selling prices were $765 340, $875 900 and $655 000. What was the total price of all three houses?

1B **4** The City of Newcastle has 390 519 residents, while the Blue Mountains region has 78 121 residents. What is the difference in the number of residents between the two local government areas?

1C **5** Quinn sold 14 bouquets of roses today. If each bouquet has 12 roses, how many roses in total did Quinn sell?

1C **6** Tyler has saved up $165. He intends to spend the money on $15 T-shirts. How many will he be able to buy?

1D **7** Aida sold equal numbers of jam donuts and cinnamon donuts today. Jam donuts are sold in boxes of six while cinnamon donuts are sold in packets of nine. What is the smallest number of each that Aida sold?

1D **8** Find the value of the following.

a 6^3 **b** 2^7

c 8 squared **d** 11^2

1D **9** Find the value of the following.

a square root of 4 **b** $\sqrt{81}$ **c** cube root of 27 **d** $\sqrt[3]{125}$

1E **10** Use the correct inequality sign to make each statement true.

a $-5 \square 2$ **b** $-5 \square -2$ **c** $7 \square -4$ **d** $4 \square -7$

1E **11** Write each list of integers in ascending order.

a 24, −40, 20, −14, 16, −22, −24

b −17, 21, −27, −35, 0, 9, 63

c −6, −66, 86, 6, −26, −16, −46

d 65, −87, 3, −300, 190, 45, −409

1F **12** Calculate:

a −5 + (−4) **b** 2 + (7) **c** 22 + (−34) **d** −50 + (69)

1F **13** Tamara's bank account shows a balance of −$28. If she deposits $150, what is her new account balance?

1F **14** Overnight, the minimum temperature in Alice Springs was −3°C. By 2 pm, the temperature had climbed to a maximum of 18°C. Calculate the difference between the minimum and maximum temperatures.

1F **15** Calculate:

a −8 + 7 **b** −4 − 6 **c** 17 − 25 **d** −44 + 34

e −66 + 66 **f** −50 − 50

1G **16** Calculate:

a 7 × (−8) **b** −5 × (−9) **c** −36 ÷ 4 **d** −100 ÷ (−20)

e −12 × 0 **f** 4 × (−15) **g** $\frac{18}{-6}$ **h** $\frac{-42}{-3}$

1G **17** Calculate:

a −6 × 3 × (−10) **b** [20 ÷ (−4)] × 7 **c** [−18 ÷ (−3)] × 2 **d** −5 × (−2) × (−4)

1G **18** Calculate:

a $(-2)^5$ **b** $(-3)^2$ **c** $(-1)^9$ **d** $(-10)^4$

1H **19** Calculate:

a $33 + (-40) \times 3 \div 10$

b $75 - 4 \times 11 + 6^2 \div (-4)$

c $-10 \times 7 - 5 \times [(-4)^2 - 4]$

d $-120 \div [-9 + (4 \times 8 - 11)]$

1H **20** Calculate:

a $-15 \times (-4 + 16)$

b $(-4)^2 - (-3)^3$

c $[-6 \times 7 + (-10 + 7)^2] \div 3$

d $[-1 + (-2)^2 + (-3)^3] \div 3$

Analysis

1 Jez is the elephant keeper at the zoo. He has two elephants to take care of: Mali and Man Jai. Mali weighs $3^3 \times 10^2$ kg while Man Jai weighs $2 \times 3^2 \times 10^2$ kg.

a What is the weight of each elephant as a basic numeral?

b What is the difference in weight between the two elephants?

c What is the sum of the weights of the two elephants?

d What is the HCF between the weight of the elephants?

Jez has to divide the food for the elephants according to their weight.

e The elephants need about one-tenth of their weight in food. How much food does Jez have to provide?

f How much food does each elephant need?

Man Jai gets sick and stops eating for a few days. As a result, he loses 5 kg each day. Jez writes the following expression to show Man Jai's weight after his illness: $2 \times 3^2 \times 10^2 - 5^n$ where n is the number of days he is sick. For example, if Man Jai is sick for 2 days, then Jez calculates his weight as $2 \times 3^2 \times 10^2 - 5^2$.

g Calculate Man Jai's weight after his illness using Jez's expression for 1, 2 and 3 days.

h Explain the problem with Jez's expression.

i Rewrite the expression so that it gives the correct weight of Man Jai after his illness.

2 The population figures (rounded to the nearest 100) for each state and territory of Australia at the end of June 2020 are displayed in this table.

State or territory	Population
New South Wales	8 164 100
Victoria	6 694 900
Queensland	5 174 400
Western Australia	2 661 900
South Australia	1 769 300
Tasmania	540 600
Australian Capital Territory	431 100
Northern Territory	246 000

a Determine the actual difference in population between NSW and Victoria.

b Round each population figure to the leading digit.

Use your answers from part **b** to answer the following questions.

c Estimate the difference in population between NSW and Victoria. Compare this to part **a**.

d Estimate the population of Australia.

3 The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Minimum temperature (°C)	1	−2	−2	−3	−5	−2	−1
Maximum temperature (°C)	8	5	9	4	−1	5	5

a Which day had:

i the highest temperature

ii the lowest temperature?

b Calculate the difference between the minimum and maximum temperatures for each day.

c Which day had the biggest range of temperatures?

d Calculate the average of the minimum and maximum temperatures.

e What is the difference between the average minimum and average maximum temperatures?

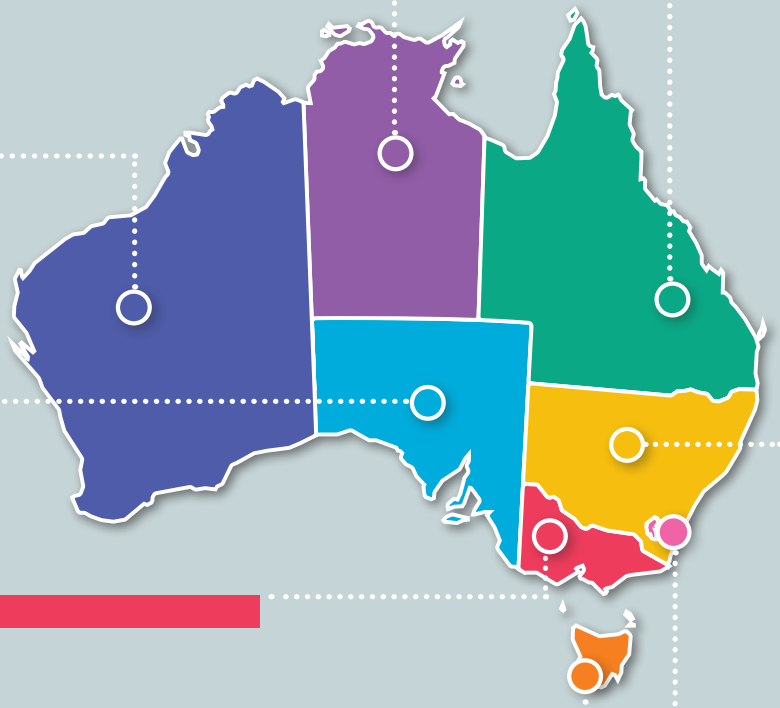
Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Round whole numbers ☐ Estimate results of simple calculations ☐ Assess the accuracy of estimations	☐ Go back to Topic 1A Rounding and estimating
☐ Use mental strategies for addition and subtraction ☐ Use the addition and subtraction algorithms for large numbers	☐ Go back to Topic 1B Adding and subtracting whole numbers
☐ Use mental strategies for multiplication ☐ Calculate products and quotients using the multiplication and division algorithms	☐ Go back to Topic 1C Multiplying and dividing whole numbers
☐ Find the lowest common multiple and the highest common factor of two numbers ☐ Evaluate squares and cubes of numbers ☐ Evaluate square roots and cube roots	☐ Go back to Topic 1D Multiples, factors, indices and roots
☐ Compare the value of positive and negative numbers ☐ Use negative numbers to represent real-life situations	☐ Go back to Topic 1E Negative integers
☐ Solve problems involving addition and subtraction of positive and negative numbers	☐ Go back to Topic 1F Adding and subtracting integers
☐ Solve problems involving multiplication and division of negative numbers ☐ Solve problems involving indices of negative numbers	☐ Go back to Topic 1G Multiplying and dividing integers
☐ Order operations in calculations involving two or more operations ☐ Evaluate expressions including integers using the order of operations	☐ Go back to Topic 1H Order of operations

Contact us

For more information about this series please contact a secondary education consultant in your state or territory:



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