MATHEMATICS

OXFORD MATHS 7 STAGE 4

SAMPLE CHAPTER

UNCORRECTED PAGE PROOFS

To learn more about the series, visit: oup.com.au/oxford-maths-nsw

Or book an appointment with your local sales consultant at oup.com.au/contact

obook pro

SERIES CONSULTANT:
KAREN BELLAMY

HELEN SORENSON
ALEXANDER BLANKSBY
JENNIFER NOLAN
MELANIE KOETSVELD
SONJA STAMBULIC
ROBERT BELL

NSW CURRICULUM

OXFORD

3 Fractions and ratios

Index

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Benchmark fractions
- ✔ Adding and subtracting whole numbers
- ✔ Multiplying and dividing whole numbers
- ✔ Highest common factor
- ✔ Number lines

Curriculum links

- Represent and operate with fractions, decimals and percentages to solve problems (MA4-FRC-C-01)
  - Compare fractions using equivalence
  - Order and compare the value of fractions, decimals and percentages
  - Solve problems that involve the addition and subtraction of fractions
  - Solve problems that involve the multiplication and division of fractions and decimals
  - Represent one quantity as a fraction, decimal or percentage of another, with and without the use of digital tools
- Solve problems involving ratios and rates, and analyse distance–time graphs (MA4-RAT-C-01)
  - Recognise and simplify ratios
  - Solve problems involving ratios

3A Fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify fractions from images, statements and number lines
- ✔ express one quantity as a fraction of another.

Inter-year links

Support — Fractions

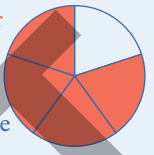
Year 8 — 2A Fractions

Fractions

- A **fraction** is a part or a portion of a whole, or multiple wholes.

$\frac{4}{5}$ ← the **numerator** shows the number of equal parts of the whole

← the **denominator** shows the total number of equal parts in one whole



- **Proper fractions** have a **numerator** that is less than the **denominator**.
  Some examples are $\frac{3}{5}, \frac{2}{6}, \frac{11}{15}$.

Proper fraction

Numerator < Denominator

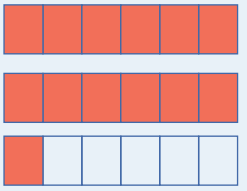
$\frac{2}{6}$ →

- **Improper fractions** have a numerator that is greater than or equal to the denominator.
  Some examples are $\frac{13}{6}, \frac{5}{5}, \frac{31}{8}$.
- **Mixed numbers** consist of both a whole number and a proper fraction.
  For example, $2\frac{1}{6} = 2 + \frac{1}{6}$.

Improper fraction

Numerator ≥ Denominator

$\frac{13}{6}$ →



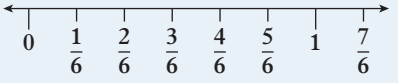
Mixed number

← $2\frac{1}{6}$

- Whole numbers can be written as fractions:
  - → where the denominator is 1,
    for example, $5 = \frac{5}{1}, 2 = \frac{2}{1}, 100 = \frac{100}{1}$
  - → or the denominator divides into the numerator without a remainder,
    for example, $\frac{2}{2} = 1, \frac{9}{3} = 3, \frac{28}{7} = 4$.

Fractions on a number line

- Fractions can be written on a number line.
  - → Number lines can be used to compare fractions.

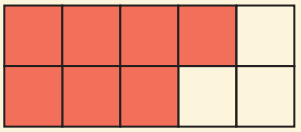


Fractions of a set

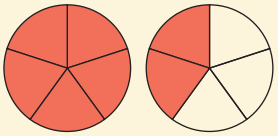
- Fractions can show the proportion or number of a group of objects within a larger set of objects.
  - → A **set** is a defined group of objects or numbers.
    For example, here is a set of 5 counters. There are 2 blue counters, so $\frac{2}{5}$ of the counters are blue.

Example 3A.1 Identifying fractions from a diagram

a The rectangle represents a whole. What fraction is shaded of the whole shape?



b Each circle represents a whole. What fraction is shaded of the whole shape?



THINK

1. Count the number of equal parts in the whole shape.
2. Count the number of shaded parts in the shape.
3. Write the number of shaded parts as the numerator, and the number of parts in each whole as the denominator.

WRITE

a The shape is cut into 10 parts.
The shape has 7 shaded parts.
The fraction of the shape that is shaded is $\frac{7}{10}$.

b The shape is cut into 5 parts. There are two shapes.
There are 7 shaded parts.
The fraction of the shape that is shaded is $\frac{7}{5}$.

Example 3A.2 Identifying fractions from a number line

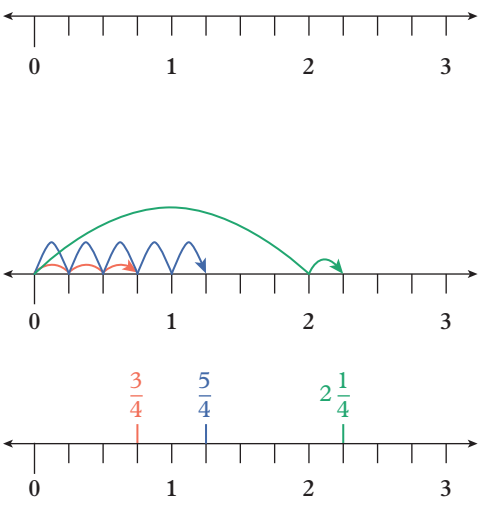
Label the fractions $\frac{3}{4}$, $\frac{5}{4}$, $2\frac{1}{4}$ on a number line.

THINK

1. Draw a number line from 0 to 3, making sure the smallest and largest fractions are between the numbers on the scale.

   Divide the number line into segments of equal lengths; the number of segments between two whole numbers should be the same as the denominator. Divide each section between two whole numbers into 4 parts.
2. Find the location of the fractions on the number line by counting up to the fraction from 0.
3. Mark and label the fractions on the number line.

WRITE



Example 3A.3 Identifying fractions from a set

Identify the fraction that is described by the following statement.

There are 17 correct answers on a quiz containing 25 questions.

THINK	WRITE
1 Determine the total number of objects in the set. There are 25 questions in the quiz.	There are 25 questions in the quiz.
2 Determine the number of objects in the group within the set. There are 17 correct answers in the quiz.	There are 17 correct answers in the quiz.
3 Write the number of objects in the group within the set as the numerator, and the number of objects in the set as the denominator.	$\frac{17}{25}$ of the answers to the quiz were correct.

Helpful hints

- ✔ Although fractions are made up of a numerator and a denominator, a fraction is just one value.
- ✔ Be careful when selecting the important information in a question. Try to determine the total number of objects in the set first. Make sure you write this as the denominator.

ANS p515

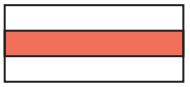
Exercise 3A Fractions

1(1st, 2nd column), 2–9, 10(b, d, f), 12–14, 15(a, b, c) | 1(3rd, 4th column), 2, 4–5, 7–9, 10, 12, 15–17, 18(a, b, c) | 1(4th column), 2, 5(e–h), 8, 10, 12, 15, 18–20

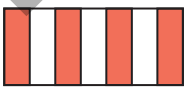
UNDERSTANDING AND FLUENCY

3A.1 **1** What fraction of each shape is shaded?

a



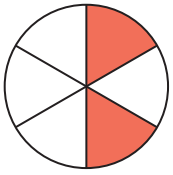
b



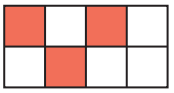
c

d

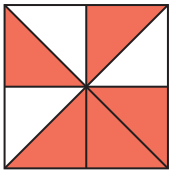
e



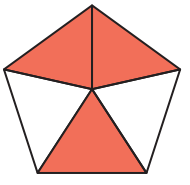
f



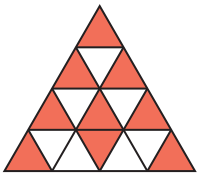
g



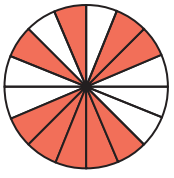
h



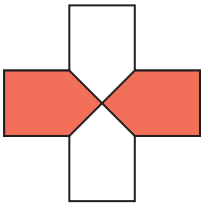
i



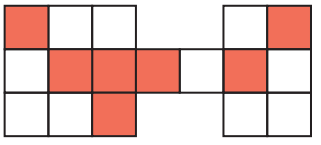
j



k



l

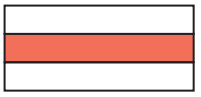


2 What fraction of each shape in question **1** is *not* shaded?

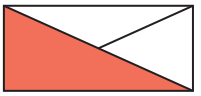
3 Which diagram correctly displays that $\frac{1}{3}$ of the shape has been shaded? Provide a reason for your selection.

A

B



C



4 Consider the following fractions: $\frac{5}{2}, 2\frac{3}{5}, \frac{7}{8}, 1\frac{5}{9}, \frac{17}{2}, \frac{16}{31}, \frac{29}{15}, 12\frac{1}{7}, \frac{13}{22}$.

a Which of the above fractions are proper fractions?

b Which of the above fractions are improper fractions?

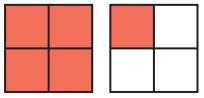
c Which of the above fractions are mixed numbers?

5 If one whole shape represents 1 whole, write the number that is represented by the shading in each diagram as:

i an improper fraction

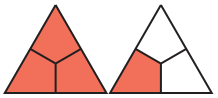
ii a mixed number.

a



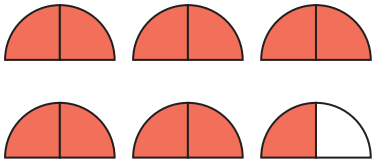
b

c

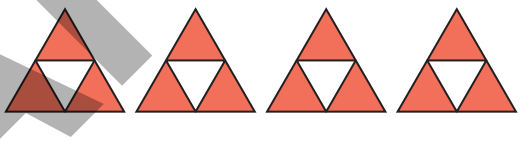


d

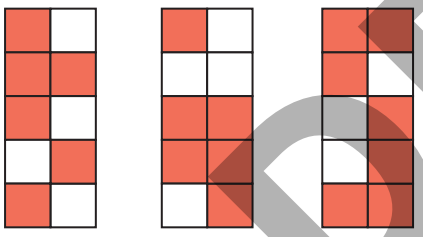
e



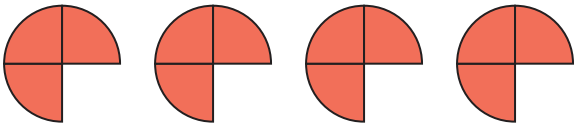
f



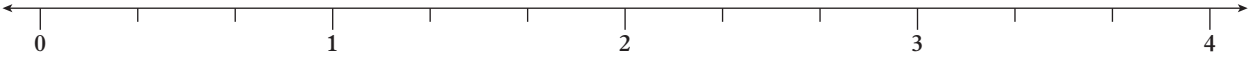
g



h



6 The number line below has been divided into four equal intervals from 0 to 4. Each interval has been further subdivided into three equal parts.



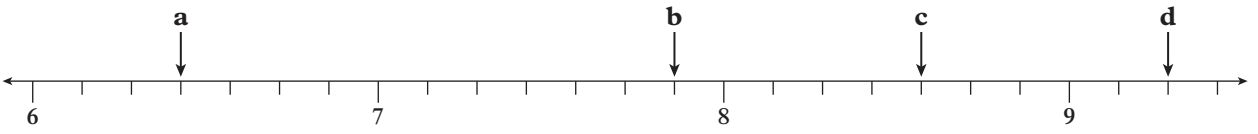
a What fraction of an interval is each part?

b Label each mark on the scale with the value it represents.

3A.2 **7** Label the fractions $\frac{2}{5}, \frac{5}{5}, \frac{8}{5}, \frac{13}{5}$ and $\frac{17}{5}$ on a number line.

8 Label the fractions $7\frac{3}{4}, \frac{36}{4}, 9\frac{2}{4}, \frac{33}{4}$ and $\frac{29}{4}$ on a number line.

9 Write down the values shown for **a–d** on the number line.



UNDERSTANDING AND FLUENCY

3A.3 **10** Identify the fraction that is described by each statement.

a 12 correct answers on a quiz containing 23 questions

b the number of months consisting of 31 days in a year

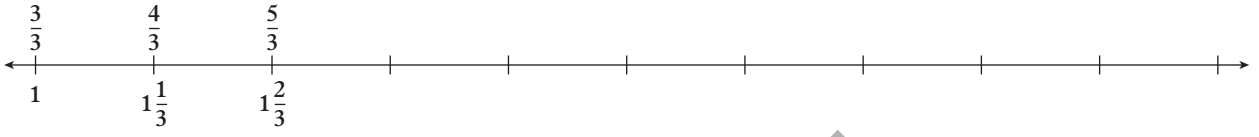
c $23 change from a $50 note

d a basket containing 41 balls, where 8 are red

e 17 episodes of a 22-episode series that have been watched

f 21 slices of pizza where each slice is one-eighth of a pizza

11 The number line below has been divided into equal intervals. The first three points have been labelled using improper fractions and mixed numbers.



a Complete the marked number line using both improper fractions and mixed numbers.

b Write down the fraction that is:

i the number that is $\frac{1}{3}$ to the right of $\frac{5}{3}$

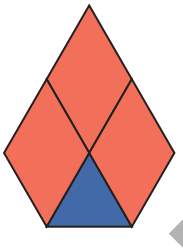
ii the improper fraction that is $\frac{2}{3}$ to the left of 4

iii the mixed number that is 1 whole to the right of $\frac{7}{3}$.

PROBLEM SOLVING AND REASONING

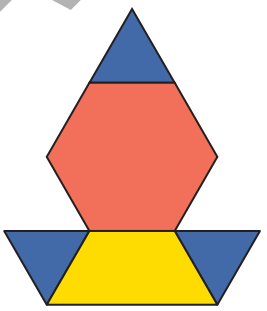
12 If the whole shape is 1 whole, determine the fraction of each colour. Hint: Can you find the blue triangles inside the other shapes?

a

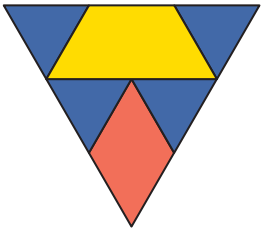


b

c



d



13 What is the difference between a proper and an improper fraction?

14 What is the difference between an improper fraction and a mixed number?

15 There are 60 minutes in 1 hour. What fraction of an hour is:

a 30 minutes

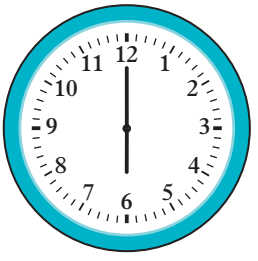
b 15 minutes

c 17 minutes

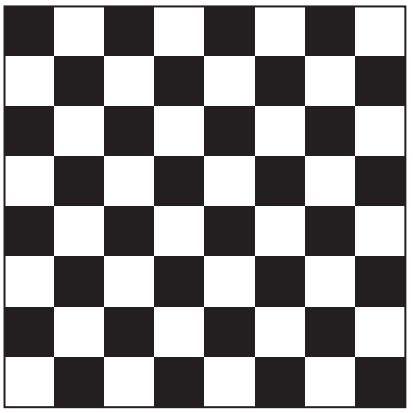
d 31 minutes

e 119 minutes (as an improper fraction)

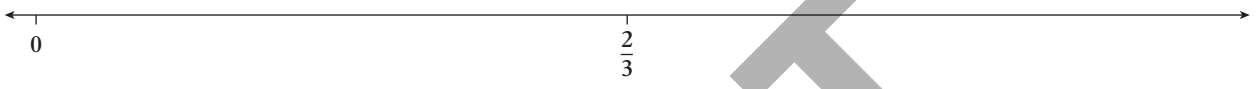
f 311 minutes (as a mixed number)?



16 Determine what fraction of the shape is shaded. Explain your reasoning.



17 a Here is a number line with 0 and $\frac{2}{3}$ marked. Copy the number line and mark where 1 should be positioned. Explain how you positioned 1.



b Here is a number line with 0 and $\frac{5}{4}$ marked. Copy the number line and mark where 1 should be positioned. Explain how you positioned 1.

18 Setting up a number line for fractions with large denominators is hard to do accurately without grid paper or a ruler. We can make it easier for denominators that are composite numbers by dividing the line into smaller segments. For example, to divide a number line into twelfths we can first divide it into halves, then quarters, and finally twelfths.

a Draw a number line from 0 to 1. Mark $\frac{6}{12}$ halfway between 0 and 1.

b Mark $\frac{3}{12}$ halfway between 0 and $\frac{6}{12}$, and mark $\frac{9}{12}$ halfway between $\frac{6}{12}$ and 1.

c Mark thirds between each marking. These are the remaining twelfths.

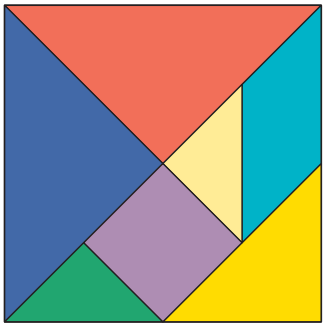
A number line can be marked for other denominators in similar ways.

d By repeatedly halving, mark a number line between 2 and 3 into sixteenths.

e Explain how you could mark a number line from 0 to 1 into fifteenths.

19 I am a fraction. My denominator has exactly three factors, and my numerator is the cube of a prime number. The difference between my denominator and my numerator is 1. What fraction am I?

20 The puzzle on the right is called a tangram. It is made up of two large triangles, one medium size triangle, two small triangles, one square and one parallelogram.



a What fraction of the whole tangram is the small triangle?

b What fraction of the whole tangram is the medium triangle?

c What fraction of the whole tangram is the large triangle?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Identifying fractions

Investigation Apple pies

Topic quiz 3A

PROBLEM SOLVING AND REASONING

CHALLENGE

3B Equivalent fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ recognise equivalent fractions on a fraction wall and number line
- ✔ identify and find equivalent fractions
- ✔ simplify fractions.

Inter-year links

Support — Equivalent fractions

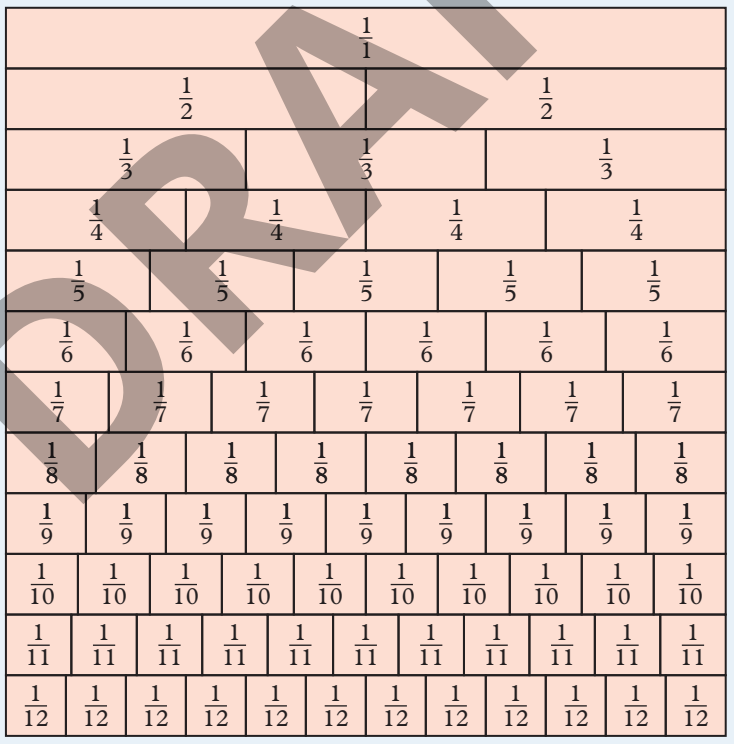
Year 8 — 2A Fractions

Equivalent fractions

- **Equivalent fractions** are fractions that have the same numerical value. They mark the same point on the number line.

  For example, $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ and $\frac{6}{12}$ all mark the same point on the number line, so they are equivalent fractions.
- A fraction is equivalent to 1 if the numerator and the denominator are the same.

  For example, $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots, \frac{10}{10}, \ldots, \frac{43}{43}, \ldots, \frac{101}{101}$ are all equivalent to 1.
- A **fraction wall** is a visual representation to help compare and identify fractions, set out in the form of a wall.

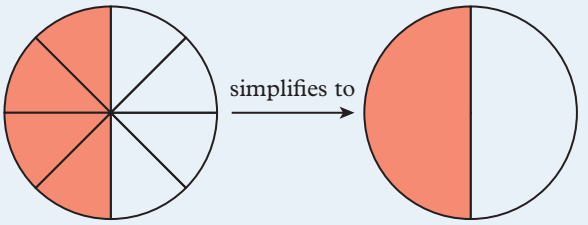


- An equivalent fraction is produced by multiplying or dividing the numerator and the denominator of a fraction by the same value.

$$\frac{2}{3} \overset{\times 5}{=} \frac{10}{15} \qquad \frac{12}{10} \overset{\div 2}{=} \frac{6}{5}$$

Simplifying fractions

- Fractions in their **simplest form** have a highest common factor of 1 between the numerator and the denominator.
  For example, $\frac{3}{4}$ is in its simplest form, as the highest common factor (HCF) between 3 and 4 is 1.



- To find an equivalent fraction in its simplest form:
  - → cancel out common factors until the only common factor between the numerator and the denominator is 1,

    for example, $\frac{36}{72} = \frac{\cancel{36}^{18}}{\cancel{72}^{36}}$
    $= \frac{\cancel{18}^{3}}{\cancel{36}^{6}}$
    $= \frac{\cancel{3}^{1}}{\cancel{6}^{2}}$
    $= \frac{1}{2}$
  - → find the HCF of the numerator and the denominator and then divide the numerator and the denominator by the HCF,

    for example, the HCF of 36 and 72 is 36, so $\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}}$
    $= \frac{1}{2}$.
- For mixed numbers, only simplify the fraction part; the whole number stays the same.

  For example, $2\frac{12}{24} = 2\frac{\cancel{12}^{1}}{\cancel{24}^{2}}$
  $= 2\frac{1}{2}$.

Example 3B.1 Writing equivalent fractions

Complete each set of equivalent fractions.

a $\frac{14}{8} = \frac{}{4}$ **b** $\frac{5}{13} = \frac{15}{}$

THINK

1 Find a relationship between the two known numerators and denominators. Consider if the equivalent fraction has been found by using multiplication or division.

2 Use the same relationship to obtain the missing numerator or denominator of the second fraction.

WRITE

a $\frac{14}{8} = \frac{}{4}$ (÷ 2)

$\frac{14}{8} = \frac{7}{4}$ (÷ 2)

b $\frac{5}{13} = \frac{15}{}$ (× 3)

$\frac{5}{13} = \frac{15}{39}$ (× 3)

Example 3B.2 Simplifying fractions using repeated division

Write each fraction in its simplest form using repeated division.

a $\frac{80}{144}$ **b** $3\frac{54}{72}$

THINK

a **1** Find a common factor of 80 and 144. They are both even numbers, so 2 is a common factor.

2 Divide both the numerator and the denominator by 2.

3 Find another common factor and repeat the process. Repeat until the only common factor is 1.

b **1** Find a common factor of 54 and 72. They both have digit sums that are multiples of 3, so 3 is a common factor.

2 Leave the whole number as it is and divide both the numerator and the denominator by 3.

3 Find another common factor and repeat the process. Repeat until the only common factor is 1.

WRITE

a $\frac{80}{144} = \frac{\cancel{80}^{40}}{\cancel{144}^{72}}$

$= \frac{\cancel{40}^{20}}{\cancel{72}^{36}}$

$= \frac{\cancel{20}^{10}}{\cancel{36}^{18}}$

$= \frac{\cancel{10}^{5}}{\cancel{18}^{9}}$

$= \frac{5}{9}$

b $3\frac{54}{72} = 3\frac{\cancel{54}^{18}}{\cancel{72}^{24}}$

$= 3\frac{\cancel{18}^{3}}{24^{4}}$

$= 3\frac{3}{4}$

Example 3B.3 Simplifying fractions using the HCF

Write each fraction in its simplest form using the HCF.

a $\frac{21}{49}$ **b** $2\frac{45}{65}$

THINK

a **1** Find the highest common factor (HCF) of the numerator and the denominator.

2 Divide both the numerator and the denominator by the HCF.

b **1** Find the HCF of the numerator and the denominator of the fraction component.

2 Leave the whole number as it is and divide both the numerator and the denominator by the HCF.

WRITE

a Factors of 21: 1, 3, (7), 21

Factors of 49: 1, (7), 49

HCF = 7

$\frac{21}{49} = \frac{\cancel{21}^{3}}{\cancel{49}^{7}}$

$= \frac{3}{7}$

b Factors of 45: 1, 3, (5), 9, 15, 45

Factors of 65: 1, (5), 13, 65

HCF = 5

$2\frac{45}{65} = 2\frac{\cancel{45}^{9}}{\cancel{65}^{13}}$

$= 2\frac{9}{13}$

Helpful hints

- ✔ Remember that 'equivalent' is another way of saying 'equal to'.
- ✔ Don't be fooled by fractions that have odd or prime numbers; sometimes they can be simplified. For example, $\frac{3}{15} = \frac{1}{5}$.
- ✔ Use the divisibility rules from Chapter 2 to help you find common factors. The larger the common factor, the faster you'll find the equivalent fraction in its simplest form.

ANS p516

Exercise 3B Equivalent fractions

△ 1(a, c, e), 2–9, 11–13 ■ 3–6, 7(b, d), 8, 10, 11, 14–15(a, b) ⬟ 3(e, f), 4(b, d, f), 5(b, d, f), 7(d), 8, 11, 13–17

1 Using the fraction wall in the theory, determine all of the fractions on the wall equivalent to:

a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{3}{12}$ **d** $\frac{2}{5}$ **e** $\frac{2}{3}$ **f** $\frac{7}{7}$

3B.1 **2** Complete each set of equivalent fractions.

a $\frac{4}{9} = \frac{}{36}$ **b** $\frac{24}{8} = \frac{12}{}$ **c** $2\frac{5}{11} = 2\frac{}{33}$

d $\frac{42}{70} = \frac{}{10}$ **e** $\frac{144}{48} = \frac{}{12}$ **f** $3\frac{2}{8} = 3\frac{}{72}$

g $\frac{2}{3} = \frac{}{9} = \frac{}{90}$ **h** $\frac{5}{6} = \frac{}{18} = \frac{20}{}$ **i** $\frac{2}{5} = \frac{10}{} = \frac{}{15}$

3 Complete each set of equivalent fractions.

a $5\frac{1}{13} = 5\frac{}{39} = 5\frac{5}{}$

b $\frac{}{10} = \frac{2}{20} = \frac{}{30} = \frac{8}{}$

c $\frac{8}{} = \frac{24}{21} = \frac{}{35} = \frac{}{77}$

d $3\frac{3}{5} = 3\frac{}{30} = 3\frac{9}{} = 3\frac{}{70}$

e $\frac{1}{12} = \frac{6}{} = \frac{8}{} = \frac{}{144} = \frac{15}{} = \frac{}{240}$

f $\frac{2}{3} = \frac{}{6} = \frac{10}{} = \frac{}{30} = \frac{}{75} = \frac{400}{}$

3B.2 **4** Write each fraction in its simplest form using repeated division.

a $\frac{10}{16}$ **b** $\frac{28}{36}$ **c** $\frac{63}{56}$ **d** $2\frac{12}{30}$ **e** $5\frac{33}{36}$ **f** $6\frac{15}{75}$

3B.3 **5** Write each fraction in its simplest form using the HCF.

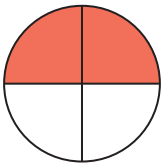
a $\frac{11}{33}$ **b** $\frac{30}{48}$ **c** $\frac{116}{94}$ **d** $7\frac{20}{45}$ **e** $4\frac{56}{64}$ **f** $3\frac{16}{54}$

6 List the fractions that are equivalent.

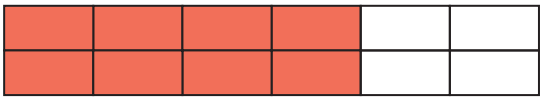
$\frac{1}{3}$	$\frac{5}{10}$	$\frac{12}{15}$	$\frac{6}{18}$	$\frac{17}{25}$
$\frac{30}{45}$	$\frac{16}{48}$	$\frac{18}{54}$	$\frac{20}{60}$	$\frac{52}{156}$

7 What fraction of each shape is shaded? Simplify your answers.

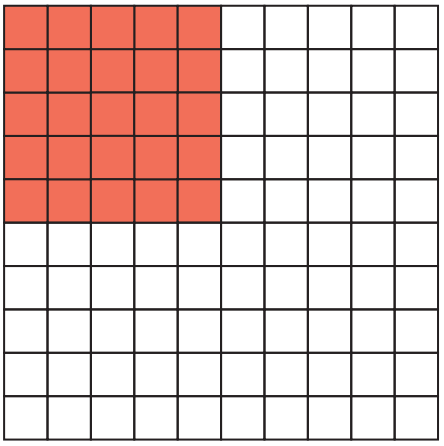
a



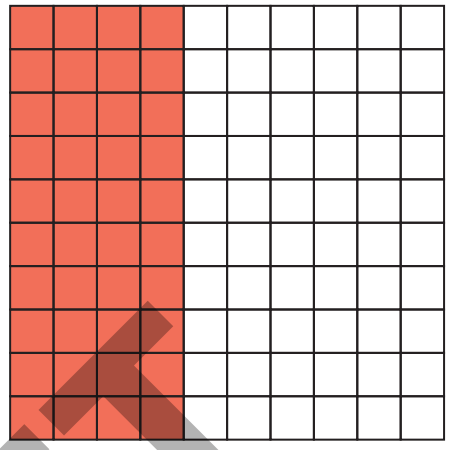
b



c



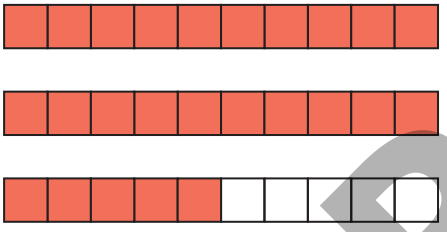
d



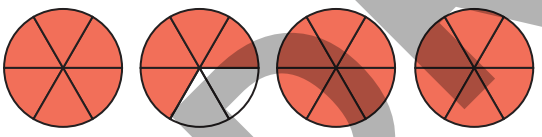
8 If each shape represents 1 whole, write the number that is represented by the shading in each diagram as:

i a simplified improper fraction **ii** a simplified mixed number.

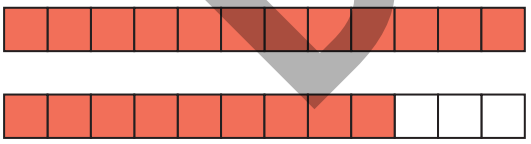
a



b



c



9 Ingrid is making toy wallabies. She has 60 buttons to use as eyes. So far she has made 13 wallabies. What fraction of the buttons does she have left? Write your answer in its simplest form.

10 Larni scored 22 of her team's 34 goals. What fraction of the team's goals did Larni score? Write your answer in its simplest form.

11 Cristiano spent 75 minutes of a 90-minute football game on the field. What fraction of time, in its simplest form, did he spend off the field?

12 Find the values of the missing numbers.

a $\frac{27}{18} = \frac{}{50}$ **b** $\frac{120}{} = \frac{48}{54}$ **c** $\frac{33}{39} = \frac{143}{}$

13 Keiynan is $\frac{5}{6}$ of the way through his 300-page book. If he reads 10 more pages, what fraction is he through the book? Write your answer in its simplest form.

14 Warragamba Dam is Sydney's largest dam. When it is full, it can hold enough water to fill the Sydney Cricket Ground 1104 times.

a If there was only enough water to fill the Sydney Cricket Ground 920 times, what fraction of the dam contains water?

b The dam's water level was below $\frac{5}{12}$ during the 2020 drought.
How many cricket grounds could be filled at that water level?

15 We can simplify a fraction by dividing the numerator and the denominator by the highest common factor (HCF). Therefore, the HCF can be determined by finding the product of all common factors used in the simplification process.

For example, $\frac{80}{144} = \frac{\cancel{80}^{40}}{\cancel{144}^{72}} = \frac{\cancel{40}^{20}}{\cancel{72}^{36}} = \frac{\cancel{20}^{10}}{\cancel{36}^{18}} = \frac{\cancel{10}^{5}}{\cancel{18}^{9}} = \frac{5}{9}$. (÷ 2, ÷ 2, ÷ 2, ÷ 2)

The HCF of 80 and 144 is $2 \times 2 \times 2 \times 2 = 16$.

Find the HCF of the following numbers by simplifying fractions.

a 42 and 315 **b** 252 and 270 **c** 3024 and 3780

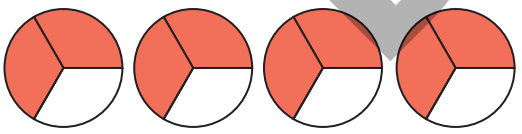
16 When simplifying by using repeated division, especially when the HCF is not obvious or not easy to determine, a useful strategy is to divide by prime numbers until there is a remainder. That is, divide by 2 until one or both of the numbers are not even, then divide by 3 until the digit sum of one or both of the numbers is not a multiple of 3, then divide by 5 until one or both of the numbers do not end in a 0 or a 5, and so on.

Simplify the following fractions using repeated division.

a $\frac{48\,600}{4800}$ **b** $\frac{121\,500}{202\,500\,000}$ **c** $\frac{196}{2058}$ **d** $\frac{161\,051}{2310}$

17 Here is a diagram showing $\frac{2}{3}$ of the whole circle shaded.

Here are four more of the same diagram. However, the fraction shaded depends on what is considered the whole.



Simplifying where possible, determine what fraction is shaded if:

a one whole is considered as one full circle

b one whole is considered as two full circles

c one whole is considered as four full circles

d one whole is considered as $\frac{1}{3}$ of a circle

e one whole is considered as $\frac{2}{3}$ of a circle

f one whole is considered as $\frac{4}{3}$ of a circle.

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet Equivalent fractions

Investigation Quilt design

Topic quiz 3B

3C Ordering fractions

Learning intentions

By the end of this topic you will be able to ...

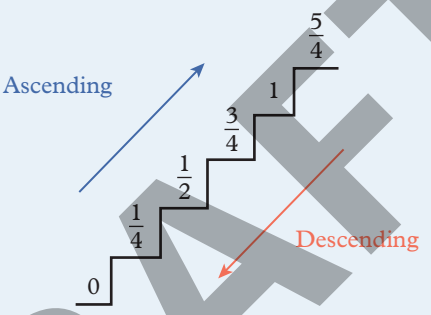
- ✔ compare fractions with the same numerator or denominator
- ✔ order fractions in ascending and descending order
- ✔ order and compare fractions using equivalent fractions.

Inter-year links

Year 8 2A Fractions

Ordering fractions

- 'Less than' is shown using the symbol '<'.
- 'Greater than' is shown using the symbol '>'.
- **Unit fractions** are fractions that have a numerator of 1. The larger the denominator of a unit fraction, the smaller the fraction.

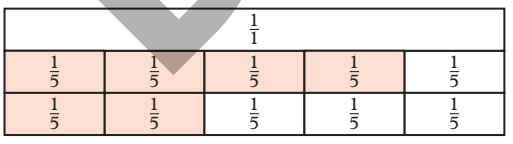


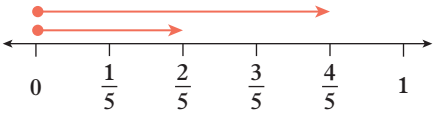
Ordering and comparing methods

Same denominator

To compare $\frac{2}{5}$ and $\frac{4}{5}$, think about having 2 parts, each one-fifth of a whole, and 4 of the same sized parts of the same sized whole.

$\frac{2}{5} < \frac{4}{5}$

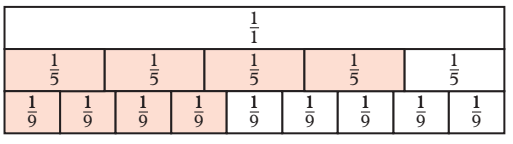


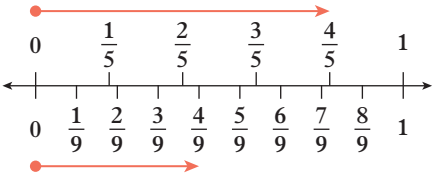


Same numerator

To compare $\frac{4}{5}$ and $\frac{4}{9}$, remember that $\frac{1}{5}$ is greater than $\frac{1}{9}$, so $\frac{4}{5}$ is 4 large parts compared to the 4 smaller parts of $\frac{4}{9}$.

$\frac{4}{9} < \frac{4}{5}$





Ordering and comparing fractions using equivalent fractions

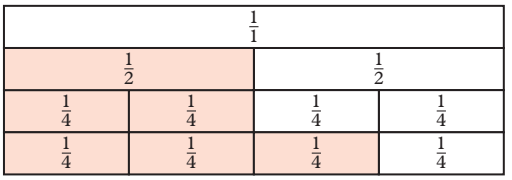
- A **common denominator** is a common multiple of the denominators of two or more fractions.

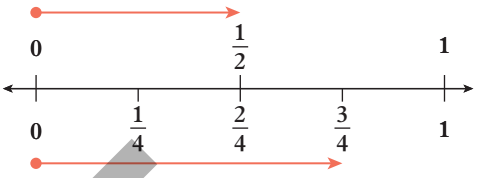
Common denominator

To compare $\frac{1}{2}$ and $\frac{3}{4}$, change $\frac{1}{2}$ to an equivalent fraction with a denominator of 4. Then $\frac{1}{2}$ can be easily compared with $\frac{3}{4}$.

$\frac{1}{2} = \frac{2}{4}$

$\frac{2}{4} < \frac{3}{4}$





- The **lowest common denominator (LCD)** is the lowest common multiple of the denominators in two or more fractions.

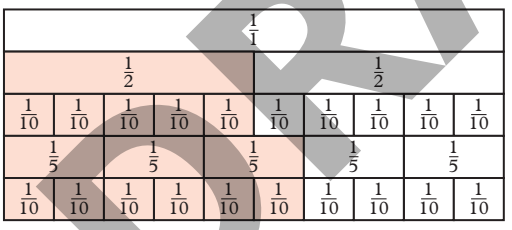
Lowest common denominator

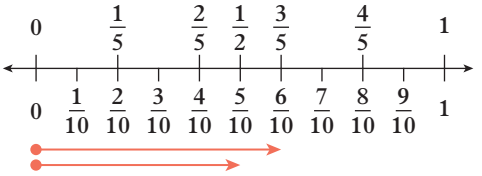
To compare $\frac{1}{2}$ and $\frac{3}{5}$, find the LCD and multiply the numerator and denominator of each fraction to get an equivalent fraction. Then compare the fractions with the same denominator.

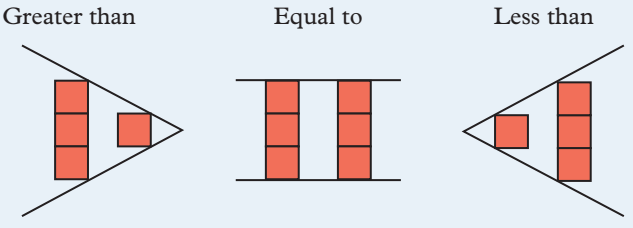
The LCD of 2 and 5 is 10.

$\frac{1}{2} \overset{\times 5}{=} \frac{5}{10}$

$\frac{3}{5} \overset{\times 2}{=} \frac{6}{10}$





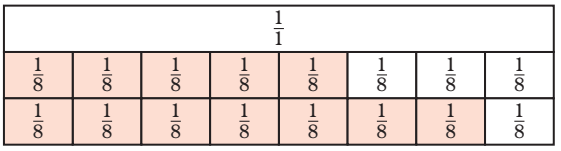


Example 3C.1 Comparing fractions with the same denominator

Which fraction is greater: $\frac{5}{8}$ or $\frac{7}{8}$? Write your answer as an inequality.

THINK

Check if the fractions have the same denominator, same numerator, or neither. These fractions have the same denominator, so $\frac{5}{8}$ is less than $\frac{7}{8}$ as it has fewer of the same parts of the whole.



WRITE

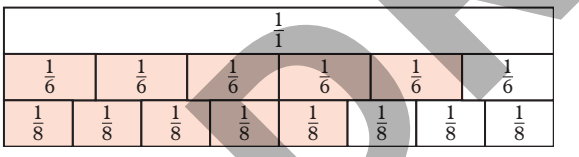
$\frac{5}{8} < \frac{7}{8}$

Example 3C.2 Comparing fractions with the same numerator

Which fraction is greater: $\frac{5}{8}$ or $\frac{5}{6}$? Write your answer as an inequality.

THINK

Check if the fractions have the same denominator, same numerator, or neither. These fractions have the same numerator, so $\frac{5}{6}$ is greater than $\frac{5}{8}$ as it has 5 greater parts of $\frac{1}{6}$ compared to 5 smaller parts of $\frac{1}{8}$.



WRITE

$\frac{5}{8} < \frac{5}{6}$

Example 3C.3 Comparing fractions with different denominators and numerators

Which fraction is greater: $\frac{5}{7}$ or $\frac{11}{14}$? Write your answer as an inequality.

THINK

1. Compare fractions by converting one fraction to an equivalent fraction that has either the same denominator or the same numerator as the other fraction. Since 14 is a multiple of 7, find an equivalent fraction to $\frac{5}{7}$ with a denominator of 14.
2. Compare the fractions with the same denominator and write the answer using the original fractions.

WRITE

$\frac{5}{7} = \frac{10}{14}$ (×2 above and ×2 below)

$\frac{10}{14} < \frac{11}{14}$, therefore $\frac{5}{7} < \frac{11}{14}$.

Example 3C.4 Ordering fractions by finding the lowest common denominator

Which fraction is greater: $\frac{5}{6}$ or $\frac{7}{9}$? Write your answer as an inequality.

THINK

1 Find the LCM of the denominators by listing the multiples of 6 and 9. Identify the lowest multiple that appears in both lists.

2 For each fraction, find the equivalent fraction with a denominator equal to the LCM.

6 multiplied by 3 is 18, so multiply the numerator and denominator of $\frac{5}{6}$ by 3.

9 multiplied by 2 is 18, so multiply the numerator and denominator of $\frac{7}{9}$ by 2.

3 Compare the fractions with the same denominator and write the answer using the original fractions.

WRITE

Multiples of 6: 6, 12, 18, 24, …

Multiples of 9: 9, 18, 27, 36, …

The LCM is 18.

So, 18 is the LCD.

$\frac{5}{6} \overset{\times 3}{=} \frac{15}{18}$ $\quad$ $\frac{7}{9} \overset{\times 2}{=} \frac{14}{18}$

$\frac{15}{18} > \frac{14}{18}$, therefore $\frac{5}{6} > \frac{7}{9}$

Helpful hints

- ✔ Remember to consider both the numerator and the denominator of fractions before comparing fractions.
- ✔ Multiplying two denominators together will find a common denominator but not always the lowest common denominator.

ANS p517

Exercise 3C Ordering fractions

△ 1–4, 5(a, c), 6(a, b), 7–11, 13, 14, 15(a, b)

□ 3(b, d, f), 5(b, d), 6(c, d), 7, 8, 9(b, c), 10(b, c), 11, 12, 15–17

⬠ 5(b, d), 6(c, d), 8, 9(c, d), 10(c, d), 11, 12, 15–20

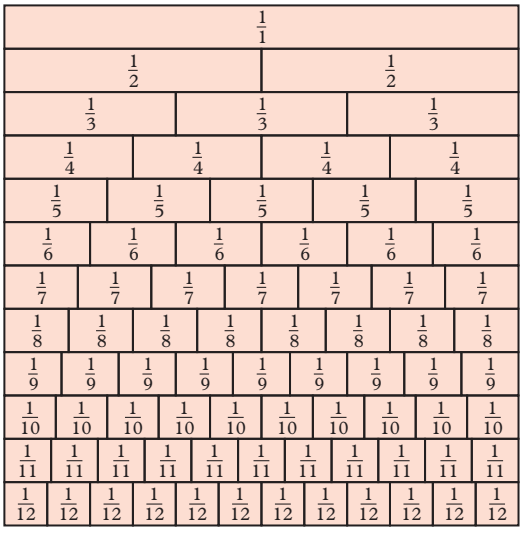
1 Use the fraction wall to decide which fraction is greater.

a $\frac{2}{3}$ or $\frac{5}{8}$

b $\frac{7}{10}$ or $\frac{9}{12}$

c $\frac{2}{12}$ or $\frac{1}{5}$

d $\frac{5}{6}$ or $\frac{7}{8}$



3C.1 **2** Identify the greater fraction in each pair. Write your answer as an inequality, with the greater fraction first.

a $\frac{3}{6}$ or $\frac{5}{6}$ **b** $\frac{7}{5}$ or $\frac{9}{5}$ **c** $\frac{93}{15}$ or $\frac{32}{15}$

d $1\frac{3}{8}$ or $1\frac{7}{8}$ **e** $3\frac{9}{20}$ or $3\frac{11}{20}$ **f** $\frac{108}{4}$ or $\frac{180}{4}$

3C.2 **3** Identify the greater fraction in each pair. Write your answer as an inequality, with the greater fraction first.

a $\frac{2}{3}$ or $\frac{2}{4}$ **b** $\frac{16}{44}$ or $\frac{16}{45}$ **c** $\frac{77}{100}$ or $\frac{77}{12}$

d $2\frac{5}{6}$ or $2\frac{5}{7}$ **e** $4\frac{13}{20}$ or $4\frac{13}{15}$ **f** $\frac{55}{17}$ or $\frac{55}{30}$

4 a Draw a number line and mark the position of each of these fractions.

$$\frac{12}{7}, \frac{10}{7}, \frac{15}{7}, \frac{5}{7}, \frac{21}{7}, \frac{32}{7}, \frac{1}{7}, \frac{31}{7}$$

b Using the number line, list the fractions in ascending order (smallest to largest) using the less than sign '<'. What do you notice?

5 Write each list of fractions in ascending order (smallest to largest) using the less than sign.

a $\frac{8}{11}, \frac{2}{11}, \frac{15}{11}, \frac{5}{11}$ **b** $\frac{16}{7}, \frac{15}{7}, \frac{6}{7}, \frac{14}{7}$ **c** $\frac{10}{5}, \frac{10}{13}, \frac{10}{15}, \frac{10}{3}$ **d** $\frac{77}{59}, \frac{77}{101}, \frac{77}{70}, \frac{77}{89}$

6 Write each list of fractions in descending order (largest to smallest).

a $\frac{1}{5}, 1\frac{2}{5}, 3\frac{2}{5}, 1\frac{4}{5}, 2\frac{1}{5}, \frac{4}{5}$ **b** $\frac{6}{5}, \frac{6}{2}, \frac{6}{6}, \frac{6}{4}, \frac{6}{3}, \frac{6}{8}$ **c** $\frac{67}{67}, \frac{1}{67}, \frac{100}{67}, \frac{76}{67}, \frac{33}{67}$ **d** $\frac{38}{40}, \frac{38}{60}, \frac{38}{2}, \frac{38}{37}, \frac{38}{19}, \frac{38}{59}$

3C.3 **7** Which fraction in each pair is greater? Write your answer as an inequality.

a $\frac{3}{6}$ or $\frac{5}{12}$ **b** $\frac{7}{10}$ or $\frac{30}{40}$ **c** $\frac{93}{15}$ or $\frac{32}{5}$

d $\frac{43}{48}$ or $\frac{7}{8}$ **e** $1\frac{9}{20}$ or $1\frac{55}{100}$ **f** $\frac{23}{2}$ or $\frac{180}{14}$

3C.4 **8** Determine which fraction is greater by finding their lowest common denominator (LCD). Write your answer as an inequality.

a $\frac{5}{6}$ or $\frac{8}{9}$ **b** $\frac{10}{20}$ or $\frac{11}{14}$ **c** $2\frac{15}{24}$ or $2\frac{14}{16}$

d $\frac{12}{7}$ or $\frac{13}{5}$ **e** $\frac{6}{17}$ or $\frac{5}{9}$ **f** $\frac{17}{16}$ or $\frac{18}{17}$

9 Write each list of fractions in ascending order.

a $\frac{7}{5}, \frac{2}{5}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{16}{5}$ **b** $\frac{5}{8}, \frac{11}{4}, \frac{2}{6}, \frac{7}{6}, \frac{5}{4}, \frac{9}{8}$ **c** $\frac{2}{9}, \frac{4}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{3}, \frac{9}{6}$ **d** $\frac{8}{15}, \frac{5}{4}, \frac{6}{3}, \frac{11}{10}, \frac{9}{20}, \frac{10}{6}$

10 Write each list of fractions in descending order.

a $\frac{3}{2}, \frac{8}{5}, \frac{7}{6}, \frac{10}{6}, \frac{5}{2}, \frac{12}{5}$ **b** $\frac{1}{8}, \frac{11}{12}, \frac{5}{6}, \frac{3}{8}, \frac{7}{12}, \frac{2}{6}$ **c** $\frac{10}{7}, \frac{9}{8}, \frac{3}{4}, \frac{3}{2}, \frac{7}{8}, \frac{6}{7}$ **d** $\frac{2}{9}, \frac{1}{3}, \frac{5}{7}, \frac{4}{3}, \frac{11}{7}, \frac{13}{9}$

11 Complete each number statement using the less than, <, or greater than, >, symbols.

a $\frac{1}{3}$ _____ $\frac{1}{4}$ **b** $\frac{2}{5}$ _____ $\frac{5}{2}$

c $4\frac{5}{6}$ _____ $\frac{23}{3}$ **d** $\frac{42}{5}$ _____ $7\frac{6}{7}$

e $\frac{7}{9}$ _____ $\frac{6}{7}$ **f** $8\frac{5}{6}$ _____ $\frac{49}{5}$

12 Without changing denominators, complete each number statement using the less than, <, or greater than, >, symbols.

a $\frac{12}{13}$ _____ $\frac{13}{14}$ **b** $\frac{4}{5}$ _____ $\frac{3}{4}$

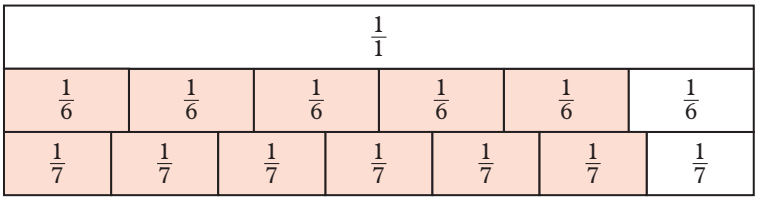
c $\frac{1}{6}$ _____ $\frac{2}{7}$ **d** $\frac{3}{9}$ _____ $\frac{3}{10}$

e $\frac{4}{19}$ _____ $\frac{4}{17}$ **f** $\frac{5}{16}$ _____ $\frac{5}{26}$

13 Philip was comparing two of his test results. He scored $\frac{22}{25}$ on the first test and $\frac{45}{50}$ on the second. On which test did Philip perform better?

14 Zach and Melanie are comparing their results after an afternoon of archery. From 35 attempts, Zach scored 20 bullseyes. Melanie scored 24 bullseyes from 40 attempts. Who hit the bullseye on the archery target with more accuracy? Justify your answer.

15 We can compare fractions that both have a numerator that is one away from a whole. The fraction with a larger denominator has smaller parts, so will be missing a smaller part.



$\frac{5}{6} < \frac{6}{7}$

Write the inequality that compares the following pairs of fractions.

a $\frac{5}{6}, \frac{8}{9}$ **b** $\frac{17}{16}, \frac{18}{17}$ **c** $\frac{100}{101}, \frac{99}{100}$ **d** $\frac{48}{47}, \frac{47}{46}$

16 The following inequalities and explanations may or may not be correct. Write the correct inequalities and give explanations.

a $\frac{5}{8} > \frac{5}{7}$ since $8 > 7$

b $\frac{70}{34} > \frac{9}{10}$ since $70 > 9$

c $\frac{17}{5} < \frac{23}{5}$ since $17 < 23$

17 Sophie and Tilly are practising shooting goals in netball. Sophie shoots 29 goals from 40 attempts and Tilly shoots 25 goals from 30 attempts. Sophie says, 'I don't want to make you feel bad, Tilly, but I am obviously the most accurate shooter since I got 29 goals and you only got 25.' Tilly replies, 'Well I don't want to make you feel bad about your maths and shooting skills, but I am the most accurate!' Which student is correct? Justify your answer.

18 Place in ascending order.

$\frac{21}{5}$	$\frac{15}{4}$	$\frac{17}{8}$	$\frac{25}{6}$	$\frac{60}{9}$	$\frac{11}{2}$	$\frac{10}{3}$

19 Place in ascending order.

$\frac{98}{99}$	$\frac{997}{999}$	$\frac{9996}{9999}$	$\frac{99\,995}{99\,999}$	$\frac{999\,994}{999\,999}$

20 A certain fraction is greater than $\frac{1}{3}$ and less than $\frac{3}{5}$. How many possible whole number values are there for the numerators if the denominator of this fraction is:

a 15 **b** 30 **c** 20?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Ordering fractions

Topic quiz
3C

3D Mixed numbers and improper fractions

Learning intentions

By the end of this topic you will be able to ...

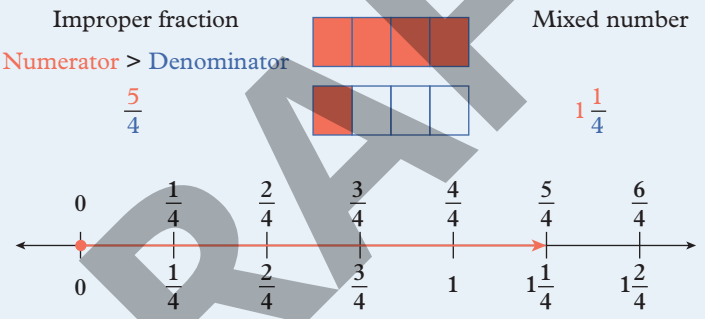
- ✔ identify mixed numbers and improper fractions on a number line
- ✔ convert improper fractions to mixed numbers
- ✔ convert mixed numbers to improper fractions.

Inter-year links

Year 8 2A Fractions

Mixed numbers and improper fractions

- Mixed numbers can be written as improper fractions. Improper fractions can also be written as mixed numbers.
- Mixed numbers consist of both a whole number and a proper fraction.
  For example, $1\frac{1}{4} = 1 + \frac{1}{4}$.



Improper fraction to mixed number

- Improper fractions show how many parts of equal size are present. Converting improper fractions to mixed numbers allows us to see how many wholes can be created from those equal parts and how many parts are left over.
- To convert an improper fraction to a mixed number:
  1. Divide the numerator by the denominator.
  2. Write down the whole number answer and place the remainder over the denominator.
  3. Simplify the fraction if possible.

$$\frac{13}{4} = 13 \div 4 = 3 \text{ remainder } 1$$
$$= 3\frac{1}{4}$$

Mixed number to improper fraction

- Mixed numbers consist of a whole number and a fraction. Converting mixed numbers to improper fractions allows us to see how many equal parts there are in the whole number and the fraction.
  1. Multiply the denominator by the whole number and add the numerator.
  2. Write the result over the denominator.

$$3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$$

Example 3D.1 Converting improper fractions to mixed numbers

Convert the improper fraction $\frac{26}{6}$ to a mixed number.

THINK

1. Divide the numerator by the denominator.
2. Write down the whole number answer and place the remainder over the denominator.
3. Simplify the fraction.

WRITE

$\frac{26}{6} = 4$ remainder 2

$4\frac{2}{6} = 4\frac{2^{1}}{6^{3}}$

$= 4\frac{1}{3}$

Example 3D.2 Converting mixed numbers to improper fractions

Convert the mixed number $3\frac{2}{7}$ to an improper fraction.

THINK

1. Multiply the denominator by the whole number and add the numerator.
2. Write the result over the denominator.

WRITE

$3\frac{2}{7} = \frac{(7 \times 3) + 2}{7}$

$= \frac{23}{7}$

Helpful hints

- ✔ An improper fraction can be simplified before or after writing it as a mixed number.
- ✔ It can be easier to simplify after writing an improper fraction as a mixed number as the numerator will be much smaller.
  For example, $\frac{12}{8} = 1\frac{4^{1}}{8^{2}} = 1\frac{1}{2}$ or $\frac{12^{3}}{8^{2}} = \frac{3}{2} = 1\frac{1}{2}$.

ANS p518

Exercise 3D Mixed numbers and improper fractions

▲ 1–3, 4(a, b), 5–6, 7(a, c, e, g), 8–11 ■ 1, 2, 4(c, d), 6, 7, 9, 10, 12, 14 ⬟ 1(e, f, k, l), 2(e, f, k, l), 4(d), 7, 9, 12–15

3D.1 **1** Convert each improper fraction to a mixed number.

a $\frac{7}{5}$ **b** $\frac{26}{3}$ **c** $\frac{25}{9}$

d $\frac{43}{10}$ **e** $\frac{75}{8}$ **f** $\frac{88}{13}$

g $\frac{22}{7}$ **h** $\frac{17}{4}$ **i** $\frac{49}{6}$

j $\frac{50}{11}$ **k** $\frac{67}{5}$ **l** $\frac{111}{2}$

3D.2 **2** Convert each mixed number to an improper fraction.

a $1\frac{2}{9}$ **b** $3\frac{1}{6}$ **c** $5\frac{8}{9}$

d $2\frac{6}{11}$ **e** $10\frac{4}{5}$ **f** $8\frac{3}{4}$

g $2\frac{3}{7}$ **h** $12\frac{1}{3}$ **i** $4\frac{5}{8}$

j $5\frac{3}{10}$ **k** $7\frac{1}{12}$ **l** $2\frac{4}{15}$

3 State whether the following statements are true or false.

a To convert an improper fraction to a mixed number, divide the numerator by the denominator, write down the whole number answer and place the remainder over the denominator.

b An improper fraction is smaller than 1.

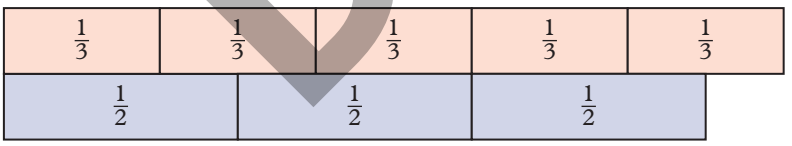
c A mixed number can have an improper fraction part.

d $\frac{45}{18} \neq 2\frac{8}{16}$

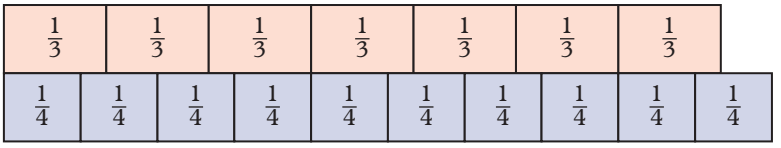
e An improper fraction can be simplified before or after converting it to a mixed number.

4 Write an inequality that compares the two fractions shown in each diagram. Provide your answers as mixed numbers and improper fractions.

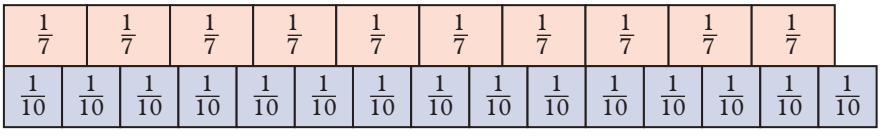
a



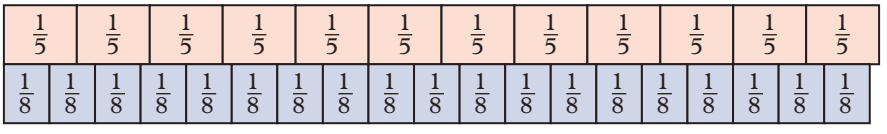
b



c



d



5 Convert each fraction to an improper fraction (if necessary) and use the symbols < or > to make a true statement.

a $2\frac{4}{7} \_\_\_\_ \frac{17}{7}$ **b** $8\frac{2}{3} \_\_\_\_ \frac{28}{3}$

c $\frac{38}{11} \_\_\_\_ 2\frac{6}{11}$ **d** $9\frac{4}{5} \_\_\_\_ \frac{51}{5}$

6 a Copy the number line and divide each interval into three equal parts. Label each of these parts using mixed numbers.

b Convert each fraction to a mixed number, simplifying where necessary, and then mark its position on the number line.

i $\frac{23}{3}$ **ii** $\frac{20}{3}$ **iii** $5\frac{2}{6}$

iv $7\frac{4}{6}$ **v** $\frac{24}{4}$ **vi** $\frac{12}{3}$

c Arrange the fractions in ascending order.

7 Convert both fractions to mixed numbers and use the < or > symbols to make a true statement.

a $\frac{28}{3} \_\_\_\_ \frac{17}{5}$ **b** $\frac{49}{6} \_\_\_\_ \frac{38}{4}$

c $\frac{13}{2} \_\_\_\_ \frac{26}{9}$ **d** $\frac{28}{5} \_\_\_\_ \frac{61}{10}$

e $\frac{42}{7} \_\_\_\_ \frac{63}{9}$ **f** $\frac{27}{6} \_\_\_\_ \frac{15}{4}$

g $\frac{32}{5} \_\_\_\_ \frac{63}{8}$ **h** $\frac{21}{2} \_\_\_\_ \frac{41}{4}$

8 Leica buys eight packets of trading cards to share equally with four of her friends.

a Write a mixed number to represent how many packets Leica and each of her friends receive.

b If each packet has 20 trading cards, how many cards does each person receive?

9 Sava is on a week-long camping trip with five friends. He has bought nine packets of marshmallows to toast on the campfire each night.

a Write a mixed number to represent how many packets of marshmallows are allocated to each person.

b If each packet contains 10 marshmallows, how many does each person receive?

10 The following conversions have mistakes. Explain the mistake and correct the calculation.

a $3\frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$

b $\frac{14}{5} = 3\frac{1}{5}$

c $6\frac{2}{3} = \frac{2}{6 \times 2 + 3} = \frac{2}{15}$

d $3\frac{4}{7} = \frac{7}{25}$

11 Darius is a chocaholic. His favourite type of chocolate comes in a block of 24 pieces. One day Darius ate $2\frac{2}{3}$ blocks of chocolate. Determine how many pieces of chocolate Darius ate.

PROBLEM SOLVING AND REASONING

12 Asha wrote the following calculation to convert $2\frac{4}{7}$ to an improper fraction:

$$\begin{aligned} 2\frac{4}{7} &= 2 + \frac{4}{7} \\ &= \frac{14}{7} + \frac{4}{7} \\ &= \frac{14 + 4}{7} \\ &= \frac{18}{7} \end{aligned}$$

a Use this method to write the following mixed numbers as improper fractions.

i $3\frac{2}{5}$ **ii** $1\frac{6}{17}$ **iii** $9\frac{1}{2}$ **iv** $9\frac{7}{10}$

b Explain how the method for converting from a mixed number to an improper fraction in the worked examples relates to the alternative method.

The reverse also works for writing an improper fraction as a mixed number:

$$\begin{aligned} \frac{18}{7} &= \frac{14 + 4}{7} \\ &= \frac{14}{7} + \frac{4}{7} \\ &= 2 + \frac{4}{7} \\ &= 2\frac{4}{7} \end{aligned}$$

c Use this method to write the following improper fractions as mixed numbers.

i $\frac{8}{3}$ **ii** $\frac{17}{2}$ **iii** $\frac{11}{6}$ **iv** $\frac{47}{10}$

d Explain how the method for converting from an improper fraction to a mixed number in the worked examples relates to the alternative method.

13 Georgina, Roisin and Vanessa decide to order large pizzas for a class lunch. Each pizza will be cut into eight equal slices. The girls estimate that each person will eat two slices and there will be a maximum of 22 people.

a Write the number of pizzas required as:

i an improper fraction

ii a mixed number.

b Will each person be able to receive the estimated two slices of pizza if five large pizzas have been ordered?

c The girls' budget does not allow them to order more than five pizzas. What is the minimum number of equal slices each pizza can be cut into, if everyone is to receive two slices?

d What fraction of a pizza will be left over if the pizzas are cut in the way suggested in part **c**?

e On the day of the lunch, three people were away and $4\frac{5}{8}$ of the pizzas were eaten. If each pizza was cut into eight slices, how many slices were eaten?

CHALLENGE

14 Write $\frac{69\,615}{31\,122}$ as a simplified mixed number.

15 There is a mixed number $a\frac{b}{c}$, where a represents a whole number, b represents the numerator and c represents the denominator of a proper fraction. After converting this mixed number into an improper fraction, the fraction becomes $\frac{ab}{c}$. For example, $9\frac{3}{10} = \frac{93}{10}$. In this case $a = 9$, $b = 3$, $c = 10$. List more fractions like this.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Mixed numbers and improper fractions

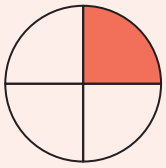
Investigation Sandwich fractions

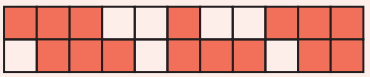
Topic quiz 3D

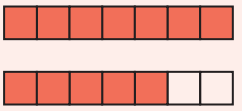
Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

3A **1** What fraction of each shape is shaded?

a 

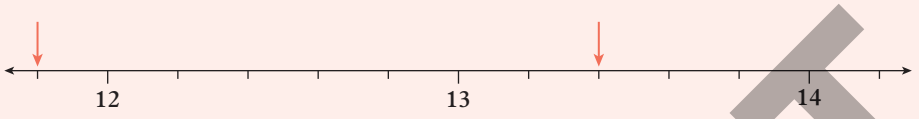
b 

c 

3A **2** Write the fractions indicated on each number line. Use mixed numbers where required.

a

b



3A **3** Write the fraction represented by each statement.

a Lachlan has collected 17 of the 20 trading cards in a set.
b Sal has surveyed 51 of the 100 people required.
c Katharine made 16 attempts at kicking a goal and got 15 in.

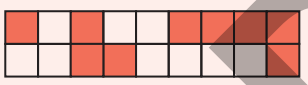
3B **4** Complete each set of equivalent fractions.

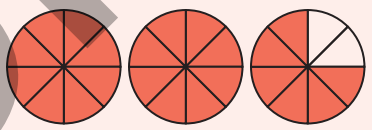
a $\frac{12}{5} = \frac{}{15}$ **b** $\frac{20}{} = \frac{4}{7}$ **c** $\frac{18}{27} = \frac{}{3} = \frac{}{6}$

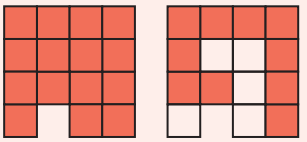
3B **5** Simplify the following fractions.

a $\frac{28}{56}$ **b** $\frac{88}{55}$ **c** $7\frac{40}{48}$

3B **6** Write the simplified fraction that the shaded section represents.

a 

b 

c 

3C **7** Write the correct inequality to compare each pair of fractions.

a $\frac{87}{121}, \frac{78}{121}$ **b** $\frac{55}{8}, \frac{55}{9}$ **c** $\frac{3}{4}, \frac{5}{6}$

3C **8** Write the following lists of fractions in ascending order.

a $\frac{9}{17}, 1\frac{5}{17}, \frac{20}{17}, \frac{1}{17}$ **b** $\frac{2}{3}, \frac{1}{6}, \frac{3}{4}, \frac{7}{12}$ **c** $5\frac{1}{8}, 5\frac{1}{3}, 5\frac{1}{4}, 5\frac{1}{6}$ **d** $\frac{15}{9}, \frac{13}{4}, \frac{17}{6}, \frac{19}{12}$

3D **9** Write these mixed numbers as simplified improper fractions.

a $2\frac{3}{5}$ **b** $6\frac{5}{7}$ **c** $12\frac{35}{40}$

3D **10** Write these improper fractions as simplified mixed numbers.

a $\frac{19}{3}$ **b** $\frac{67}{8}$ **c** $\frac{180}{21}$

3E Adding and subtracting fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ add and subtract fractions with the same denominator
- ✔ add and subtract fractions with different denominators
- ✔ add and subtract mixed numbers.

Inter-year links

Support	Adding and subtracting fractions
Year 8	2B Adding and subtracting fractions

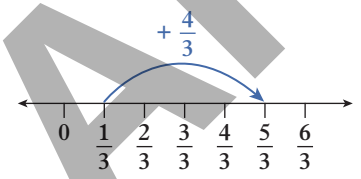
Adding and subtracting like and unlike fractions

- **Like fractions** have the same denominators and **unlike fractions** have different denominators. For example, $\frac{2}{3}$ and $\frac{4}{3}$ are like fractions, whereas $\frac{2}{3}$ and $\frac{5}{4}$ are unlike fractions.

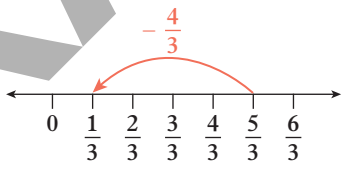
Adding and subtracting like fractions

1 Add or subtract the numerators and keep the same denominator.
2 Simplify the fraction.

$\frac{1}{3} + \frac{4}{3} = \frac{5}{3}$



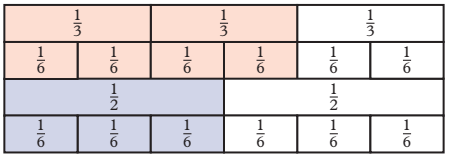
$\frac{5}{3} - \frac{4}{3} = \frac{1}{3}$



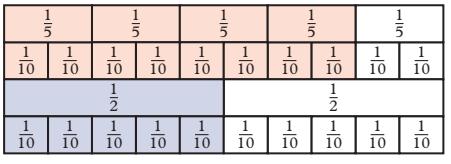
Adding and subtracting unlike fractions

1 Convert into equivalent fractions with a common denominator.
2 Add or subtract the numerators and keep the same denominator.
3 Simplify the fraction.

$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6}$
$= \frac{7}{6}$



$\frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10}$
$= \frac{3}{10}$



Adding and subtracting mixed numbers

Adding and subtracting mixed numbers

1. Convert mixed numbers into improper fractions.
2. Convert the improper fractions into equivalent fractions with a common denominator.
3. Add or subtract the numerators and keep the same denominator.
4. Convert the answer into a mixed number and simplify if possible.

$$2\frac{1}{4} + 3\frac{1}{5} = \frac{9}{4} + \frac{16}{5}$$
$$= \frac{45}{20} + \frac{64}{20}$$
$$= \frac{109}{20}$$
$$= 5\frac{9}{20}$$

$$2\frac{3}{4} - 1\frac{1}{2} = \frac{11}{4} - \frac{3}{2}$$
$$= \frac{11}{4} - \frac{6}{4}$$
$$= \frac{5}{4}$$
$$= 1\frac{1}{4}$$

- Before adding and subtracting fractions, it can be helpful to estimate the answer. Consider if the answer will be greater than or equal to $\frac{1}{2}$, 1, 2 etc. For mixed numbers, you can estimate by adding or subtracting just the whole number parts.

Example 3E.1 Adding and subtracting like fractions

Find the result of each of these.

a $\frac{3}{8} + \frac{7}{8}$ **b** $\frac{9}{14} - \frac{2}{14}$ **c** $\frac{1}{5} + \frac{3}{5} - \frac{2}{5}$

THINK

a 1 Add the numerators and keep the same denominator.

2 Simplify the fraction by dividing the numerator and denominator by the HCF.

b 1 Subtract the numerators and keep the same denominator.

2 Simplify the fraction by dividing the numerator and denominator by the HCF.

c 1 Remember the order of operations. Addition and subtraction are both the last operation to be applied, so work from left to right.

2 Simplify the fraction. The HCF of 2 and 5 is 1, so the fraction is already in its simplest form.

WRITE

a $\frac{3}{8} + \frac{7}{8} = \frac{10}{8}$
$= \frac{\cancel{10}^{5}}{\cancel{8}^{4}}$
$= \frac{5}{4}$

b $\frac{9}{14} - \frac{2}{14} = \frac{7}{14}$
$= \frac{\cancel{7}^{1}}{\cancel{14}^{2}}$
$= \frac{1}{2}$

c $\frac{1}{5} + \frac{3}{5} - \frac{2}{5} = \frac{4}{5} - \frac{2}{5}$
$= \frac{2}{5}$

Example 3E.2 Adding and subtracting unlike fractions

Find the result of each of these.

a $\frac{5}{6} + \frac{1}{2}$ **b** $\frac{11}{5} - \frac{6}{4}$

THINK

a 1 Convert into equivalent fractions with a common denominator. The LCM of 6 and 2 is 6. Change $\frac{1}{2}$ to an equivalent fraction with a denominator of 6.

2 Add the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

b 1 Convert into equivalent fractions with a common denominator. The LCM of 5 and 4 is 20. Change both fractions to equivalent fractions with a denominator of 20.

2 Subtract the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

WRITE

a $\frac{1}{2} = \frac{3}{6}$ (× 3 above, × 3 below)

$\frac{5}{6} + \frac{1}{2} = \frac{5}{6} + \frac{3}{6}$

$= \frac{8^{4}}{6^{3}}$

$= \frac{4}{3}$

b $\frac{11}{5} = \frac{44}{20}$ (× 4 above, × 4 below) $\quad \frac{6}{4} = \frac{30}{20}$ (× 5 above, × 5 below)

$\frac{11}{5} - \frac{6}{4} = \frac{44}{20} - \frac{30}{20}$

$= \frac{14^{7}}{20^{10}}$

$= \frac{7}{10}$

Example 3E.3 Adding and subtracting mixed numbers

Find the result of each of these.

a $1\frac{4}{9} + 2\frac{2}{9}$ **b** $4\frac{1}{3} - 2\frac{1}{4}$

THINK

a 1 Convert the mixed numbers into improper fractions.

2 Add the numerators and keep the same denominator.

3 Convert the answer into a mixed number and simplify if possible.

b 1 Convert the mixed numbers into improper fractions.

2 Rewrite the improper fractions with a common denominator. The LCM multiple of 3 and 4 is 12.

3 Subtract the numerators and keep the same denominator.

4 Convert the answer into a mixed number and simplify if possible.

WRITE

a $1\frac{4}{9} + 2\frac{2}{9} = \frac{13}{9} + \frac{20}{9}$ (+ above, × below)

$\frac{13}{9} + \frac{20}{9} = \frac{33}{9}$

$= 3\frac{6^{2}}{9^{3}}$ (33 ÷ 9 = 3 remainder 6)

$= 3\frac{2}{3}$

b $4\frac{1}{3} - 2\frac{1}{4} = \frac{13}{3} - \frac{9}{4}$ (+ above, × below) $\quad \frac{13}{3} = \frac{52}{12}$ (× 4 above, × 4 below) $\quad \frac{9}{4} = \frac{27}{12}$ (× 3 above, × 3 below)

$\frac{13}{3} - \frac{9}{4} = \frac{52}{12} - \frac{27}{12}$

$= \frac{25}{12}$ (25 ÷ 12 = 2 remainder 1)

$= 2\frac{1}{12}$

Helpful hints

- ✔ The key to success in adding and subtracting fractions is a common denominator. When all fractions have the same denominator, you only need to add or subtract the numerators.
  For example, $\frac{5}{6} - \frac{2}{6} \neq \frac{5-2}{6-6}$.
- ✔ Sometimes you only need to find an equivalent fraction of one of your fractions for their denominators to be the same.
  For example, $\frac{1}{4} - \frac{1}{8} = \frac{2}{8} - \frac{1}{8}$.

ANS p518

Exercise 3E Adding and subtracting fractions

△ 1–5(1st, 2nd columns), 6–9, 10(a, b), 11–13, 18(a, c)
□ 4(b, d, f, g), 5–6, 8–10, 14, 15, 17, 18(b, c, d), 19(a)
⬠ 4–5(3rd column), 6, 8(g, h), 9–10, 14, 16, 17, 18(b, d), 19–21

1 Find the result of each of these. Write your answer as a mixed number if possible.

a $1 + \frac{1}{4}$ **b** $2 + \frac{1}{2}$ **c** $\frac{3}{5} + 3$

d $12 - \frac{1}{3}$ **e** $5 - \frac{7}{12}$ **f** $13 - \frac{1}{13}$

g $2 + 4\frac{2}{3}$ **h** $3 + 2\frac{1}{2}$ **i** $4\frac{3}{5} + 3$

j $2 - 1\frac{11}{33}$ **k** $53 - 28\frac{3}{12}$ **l** $30 - 3\frac{2}{13}$

3E.1 **2** Find the result of each sum. Simplify and give your answer as a mixed number if possible.

a $\frac{1}{5} + \frac{2}{5}$ **b** $\frac{4}{7} + \frac{2}{7}$ **c** $\frac{5}{12} + \frac{3}{12}$

d $\frac{5}{18} + \frac{6}{18}$ **e** $\frac{3}{37} + \frac{16}{37}$ **f** $\frac{9}{23} + \frac{16}{23}$

g $\frac{15}{11} + \frac{2}{11} + \frac{8}{11}$ **h** $\frac{2}{13} + \frac{4}{13} + \frac{5}{13}$ **i** $\frac{23}{100} + \frac{7}{100} + \frac{14}{100}$

3 Find the result of each difference. Simplify and give your answer as a mixed number if possible.

a $\frac{11}{7} - \frac{5}{7}$ **b** $\frac{25}{21} - \frac{8}{21}$ **c** $\frac{19}{13} - \frac{5}{13}$

d $\frac{13}{15} - \frac{3}{15}$ **e** $\frac{26}{27} - \frac{25}{27}$ **f** $\frac{21}{33} - \frac{5}{33}$

g $\frac{48}{17} - \frac{13}{17} - \frac{10}{17}$ **h** $\frac{55}{3} - \frac{47}{3} - \frac{7}{3}$ **i** $\frac{21}{8} - \frac{2}{8} - \frac{15}{8}$

3E.2 **4** Find the result of each of these. Simplify your answer.

a $\frac{2}{3} + \frac{1}{6}$ **b** $\frac{2}{12} + \frac{5}{6}$ **c** $\frac{1}{2} - \frac{3}{8}$

d $\frac{3}{4} - \frac{3}{16}$ **e** $\frac{2}{15} + \frac{4}{5}$ **f** $\frac{4}{30} + \frac{7}{10}$

g $\frac{5}{8} - \frac{5}{48}$ **h** $\frac{3}{26} + \frac{1}{13}$ **i** $\frac{17}{23} - \frac{1}{69}$

5 Find the result of each of these.

a $\frac{1}{2}+\frac{2}{3}$ **b** $\frac{3}{5}-\frac{1}{4}$ **c** $\frac{3}{8}+\frac{5}{6}$

d $\frac{5}{9}-\frac{1}{2}$ **e** $\frac{3}{4}+\frac{2}{3}$ **f** $\frac{1}{3}-\frac{1}{5}$

g $\frac{4}{7}+\frac{1}{2}$ **h** $\frac{5}{6}-\frac{3}{5}$ **i** $\frac{6}{7}-\frac{1}{3}$

6 Find the result of each of these.

a $\frac{2}{3}+\frac{11}{12}+\frac{3}{4}$ **b** $\frac{3}{5}+\frac{13}{20}+\frac{12}{15}$ **c** $\frac{7}{8}+\frac{5}{3}-\frac{5}{12}$

d $\frac{5}{6}-\frac{1}{2}+\frac{3}{8}$ **e** $\frac{21}{5}+\frac{8}{3}-\frac{13}{10}$ **f** $\frac{8}{5}-\frac{11}{8}+\frac{1}{4}$

7 **a** Match the following diagrams with the correct calculation. Determine which diagram and which calculation does not have a match.

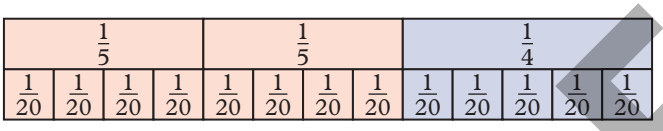
b Write the answer to each calculation. Use the diagrams to assist you.

c Write a calculation and its answer for the diagram without a match.

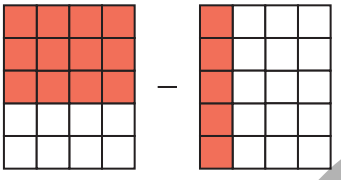
d Draw a diagram (fraction wall or number line) for the calculation without a match.

i $\frac{2}{5}-\frac{1}{4}$ **ii** $\frac{3}{5}+\frac{1}{4}$ **iii** $\frac{2}{5}+\frac{1}{4}$ **iv** $\frac{3}{5}-\frac{1}{4}$

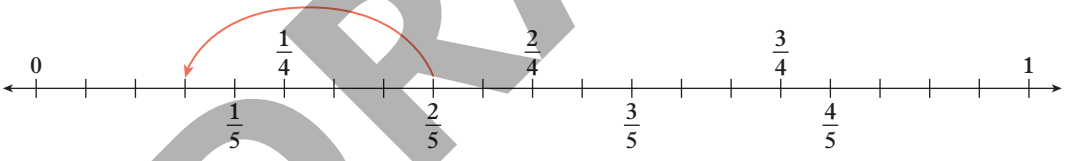
A



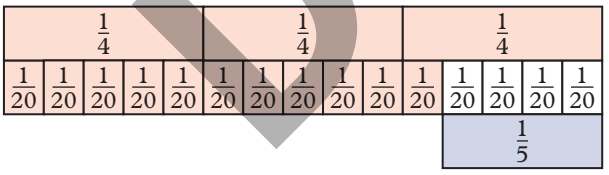
B



C



D



3E.3 **8** Find the result of each of these.

a $3\frac{2}{5}+4\frac{1}{5}$ **b** $6\frac{4}{5}-3\frac{2}{5}$ **c** $2\frac{8}{11}+1\frac{5}{11}$ **d** $8\frac{1}{9}-4\frac{4}{9}$

e $12\frac{2}{3}-2\frac{1}{3}-8\frac{2}{3}$ **f** $3\frac{2}{5}+4\frac{1}{5}+1\frac{4}{5}$ **g** $6\frac{5}{12}+2\frac{1}{12}-7\frac{11}{12}$ **h** $3\frac{1}{8}+2\frac{5}{8}-1\frac{7}{8}$

9 Find the result of each of these.

a $2\frac{2}{3}+1\frac{1}{2}$ **b** $4\frac{4}{7}-1\frac{2}{3}$ **c** $3\frac{2}{5}+2\frac{1}{4}$ **d** $5\frac{2}{3}-3\frac{4}{5}$

e $6\frac{5}{6}-4\frac{3}{10}$ **f** $3\frac{2}{14}+2\frac{2}{21}$ **g** $1\frac{5}{6}+5\frac{1}{2}$ **h** $4\frac{1}{6}-2\frac{7}{24}$

10 Find the result of each of these.

a $\frac{8}{3}+5+1\frac{1}{8}$ **b** $1\frac{1}{7}+2\frac{1}{2}-3\frac{5}{14}$ **c** $\frac{28}{5}-4\frac{1}{7}+1\frac{1}{2}$ **d** $3\frac{5}{12}-\frac{10}{5}+\frac{17}{6}$

11 Dean completed a 10 km run in $\frac{5}{7}$ of an hour, while Mark completed the run in $\frac{3}{4}$ of an hour.

a Which runner was faster?

b What was the difference between the two times?

12 A factory worker is employed to make T-shirts. On average, each T-shirt takes $3\frac{1}{2}$ minutes for cutting, $2\frac{4}{5}$ minutes for sewing and $4\frac{1}{6}$ minutes for finishing. What is the total time required to make one T-shirt?

13 Explain why the following calculations are wrong.

a $\frac{3}{4} + \frac{3}{4} = \frac{6}{8}$ **b** $\frac{3}{8} - \frac{1}{2} - \frac{1}{3} = \frac{1}{3}$

14 A group of friends organises to catch up for lunch and decides to order pizza. Each person estimates what fraction of a pizza he or she can eat. Luisa can eat $\frac{1}{3}$, Declan $\frac{1}{2}$, Nisa $\frac{3}{8}$, Tomas $\frac{2}{3}$ and Christopher $\frac{3}{2}$ of a pizza.

a How many full pizzas will need to be ordered?

b What fraction of a pizza will be left over?

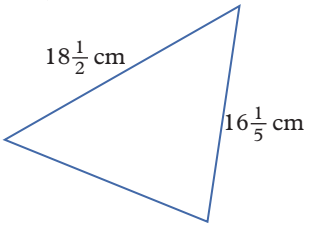
15 Laila is planning to travel overseas at the end of the year and needs to save money, so she devises a budget. She decides to allocate $\frac{2}{5}$ of her weekly pay to living expenses, $\frac{1}{3}$ to rent, $\frac{1}{8}$ to going out and the remainder to savings.

a Calculate what fraction of Laila's pay will go towards living expenses, rent and going out.

b What fraction of her pay will go towards savings? Explain your answer.

16 The total length around the outside of the triangle on the right is $49\frac{2}{5}$ cm.

The length measurements of two of the sides are known. What is the length of the unknown side?



17 While on camp, students were required to complete a 21 km bushwalk over three days. They covered $8\frac{2}{5}$ km on the first day and $7\frac{1}{3}$ km on the second day.

a What distance will they need to walk on the third day?

b How much further did they walk on the first day than on the second?

18 Complete each of these patterns by filling in the gaps. Explain how the patterns work.

a $\frac{2}{7}, \frac{5}{7}, \frac{8}{7}, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_$ **b** $10, 11\frac{2}{5}, 12\frac{4}{5}, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_$

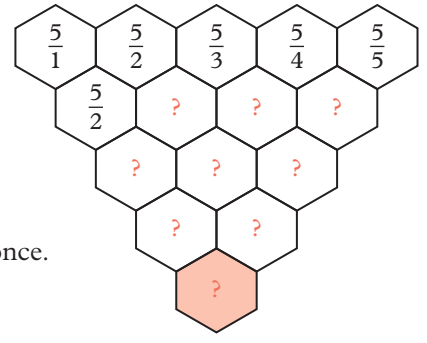
c $\frac{19}{3}, \frac{16}{3}, \frac{13}{3}, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_$ **d** $12\frac{14}{15}, 10\frac{10}{15}, 8\frac{6}{15}, \_\_\_\_, \_\_\_\_, \_\_\_\_$

19 Use a single pair of brackets to make each equation true.

a $\frac{1}{2} - \frac{5}{8} - \frac{2}{5} = \frac{11}{40}$ **b** $\frac{19}{28} - \frac{3}{7} + \frac{3}{14} = \frac{1}{28}$

20 Replace each question mark with the difference between the two fractions above it. What will the number at the bottom be?

Repeat this process with $\frac{9}{1}, \frac{9}{2}, \frac{9}{3}, \frac{9}{4}, \frac{9}{5}, \frac{9}{6}, \frac{9}{7}, \frac{9}{8}, \frac{9}{9}$ across the top. What will the number at the bottom of this triangle be?



21 Create a true statement with the form below using each digit from 1 to 9 once.

$\square\frac{\square}{\square} + \square\frac{\square}{\square} = \square\frac{\square}{\square}$

3F Multiplying fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ multiply fractions by whole numbers
- ✔ multiply fractions by fractions
- ✔ multiply mixed numbers.

Inter-year links

Year 8 — 2C Multiplying and dividing fractions

Multiplying fractions

- When multiplying fractions, the denominators do not need to be the same.
- To multiply two fractions together:

1. Multiply the numerators together.
2. Multiply the denominators together.
3. Simplify the fraction.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

- When multiplying mixed numbers, convert the mixed numbers into improper fractions, multiply across, and give the result as a mixed number.

$$1\frac{3}{5} \times 2\frac{2}{7} = \frac{8}{5} \times \frac{16}{7} = \frac{108}{35} = 3\frac{3}{5}$$

Cancelling

- When multiplying fractions, **cancelling** or dividing out common factors can simplify the process.
  → Cancelling can only be performed diagonally and vertically, not horizontally.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times \cancel{3}^{1}}{\cancel{3}^{1} \times 4} = \frac{\cancel{2}^{1}}{\cancel{4}^{2}} = \frac{1}{2}$$

Diagonally

Vertically

Multiplying by whole numbers

- Whole numbers can be written as fractions with a denominator of 1. For example, $8 = \frac{8}{1}$.
- The number 1 can be written as a fraction where the numerator and denominator are equal.
  For example, $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{100}{100}$ ...
- When multiplying any number or fraction by 1, the result remains the same.
- When asked to find a fraction 'of' a whole number or fraction, replace the 'of' with '×'.
  For example, $\frac{2}{3}$ of 20 is the same as $\frac{2}{3} \times 20$.

Example 3F.1 Multiplying fractions with simplifying

Calculate the result of $\frac{3}{5} \times \frac{2}{6}$.

THINK

1 Multiply the numerators together, and then multiply the denominators together.

2 Simplify the fraction. 6 and 30 have a common factor of 6.

WRITE

$$\frac{3}{5} \times \frac{2}{6} = \frac{3 \times 2}{5 \times 6}$$

$$= \frac{\cancel{6}^{1}}{\cancel{30}^{5}}$$

$$= \frac{1}{5}$$

Example 3F.2 Multiplying fractions with cancelling

Calculate the result of $\frac{8}{5} \times \frac{25}{12}$.

THINK

1 Look diagonally and vertically for any common factors between the numerators and denominators.

Cancel the 8 and 12 by dividing both numbers by 4. Then cancel the 25 and 5 by dividing both numbers by 5.

2 Multiply the numerators together, and then multiply the denominators together.

WRITE

$$\frac{8}{5} \times \frac{25}{12} = \frac{\cancel{8}^{2}}{\cancel{5}^{1}} \times \frac{\cancel{25}^{5}}{\cancel{12}^{3}}$$

$$= \frac{2 \times 5}{1 \times 3}$$

$$= \frac{10}{3}$$

Example 3F.3 Multiplying mixed numbers

Calculate the result of $1\frac{2}{3} \times 2\frac{7}{10}$.

THINK

1 Convert each mixed number into an improper fraction.

2 Look diagonally and vertically for any common factors between the numerators and denominators.

Cancel the 5 and 10 by dividing both numbers by 5. Then cancel the 3 and 27 by dividing both numbers by 3.

3 Multiply the numerators together, and then multiply the denominators together.

4 Convert the improper fraction to a mixed number.

WRITE

$$1\frac{2}{3} \times 2\frac{7}{10} = \frac{5}{3} \times \frac{27}{10}$$

$$= \frac{\cancel{5}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{27}^{9}}{\cancel{10}^{2}}$$

$$= \frac{1 \times 9}{1 \times 2}$$

$$= \frac{9}{2}$$

$$= 4\frac{1}{2}$$

Helpful hints

- ✔ Remember when multiplying and dividing fractions you do not need to write the fractions with a common denominator.
- ✔ Always write whole numbers as fractions before multiplying.
- ✔ Always convert mixed numbers to improper fractions before multiplying.
- ✔ Cancelling out common factors of the numerators and denominators before multiplying means you are simplifying before multiplying (which is often easier since you are working with smaller numbers).

ANS p519

Exercise 3F Multiplying fractions

△ 1–6, 8(a, c, f, h), 10–13, 15, 16(a) | □ 2(d, e, g, h), 3–4, 5(e–h), 6, 8–10, 12, 14, 16, 17 | ⬠ 2–4(g–i), 5(g, h), 6(d, e, f), 9–10, 12, 14, 16–18

UNDERSTANDING AND FLUENCY

1 Use repeated addition to calculate the following products.

a $3 \times \frac{2}{5}$ **b** $4 \times \frac{2}{9}$ **c** $6 \times \frac{1}{6}$ **d** $2 \times \frac{11}{3}$

3F.1 **2** Calculate each of these products. Leave your answer as an improper fraction if applicable.

a $\frac{1}{5} \times \frac{3}{2}$ **b** $\frac{2}{3} \times \frac{1}{7}$ **c** $\frac{11}{3} \times \frac{2}{5}$

d $\frac{1}{3} \times \frac{4}{9}$ **e** $\frac{3}{8} \times \frac{7}{2}$ **f** $\frac{11}{13} \times \frac{6}{7}$

g $\frac{1}{3} \times \frac{7}{15}$ **h** $\frac{3}{10} \times \frac{9}{7}$ **i** $\frac{9}{2} \times \frac{9}{5}$

3F.2 **3** Calculate each of these products.

a $\frac{8}{9} \times \frac{1}{4}$ **b** $\frac{2}{3} \times \frac{6}{11}$ **c** $\frac{3}{5} \times \frac{10}{13}$

d $\frac{1}{8} \times \frac{4}{17}$ **e** $\frac{7}{9} \times \frac{12}{35}$ **f** $\frac{20}{27} \times \frac{9}{15}$

g $\frac{3}{4} \times \frac{18}{24}$ **h** $\frac{9}{35} \times \frac{20}{21}$ **i** $\frac{14}{28} \times \frac{8}{13}$

3F.3 **4** Calculate each of these products. Give your answer as a mixed number if possible.

a $\frac{3}{5} \times 1\frac{4}{11}$ **b** $5\frac{2}{4} \times \frac{6}{7}$ **c** $5\frac{3}{5} \times 3\frac{1}{4}$

d $4\frac{2}{3} \times \frac{6}{2}$ **e** $\frac{7}{9} \times 3\frac{5}{7}$ **f** $5\frac{2}{3} \times 2\frac{2}{5}$

g $1\frac{3}{20} \times \frac{8}{5}$ **h** $3\frac{1}{3} \times 4\frac{1}{8}$ **i** $2\frac{4}{9} \times 3\frac{3}{16}$

5 Calculate each of these products.

a $\frac{3}{8} \times 24$ **b** $\frac{5}{11} \times 66$ **c** $\frac{12}{15} \times 4$ **d** $5 \times \frac{16}{25}$

e $9 \times \frac{1}{24}$ **f** $6 \times \frac{5}{36}$ **g** $\frac{8}{3} \times 15$ **h** $9 \times \frac{4}{81}$

6 Calculate:

a $\frac{2}{5}$ of $\frac{1}{8}$ **b** $\frac{3}{4}$ of $\frac{3}{7}$ **c** $\frac{2}{11}$ of $\frac{7}{9}$

d $\frac{9}{4}$ of $\frac{5}{11}$ **e** $2\frac{7}{13}$ of 26 **f** $5\frac{1}{2}$ of $3\frac{1}{3}$

7 Identify whether the following statements are true or false.

a When multiplying a whole number by a proper fraction, the answer is always smaller than the whole number.

b When multiplying a whole number by an improper fraction or mixed number, the answer is always smaller than the whole number.

8 When multiplying three fractions together, start by multiplying the first two fractions together, then multiply the result by the third fraction. Calculate:

a $\frac{5}{9} \times \frac{9}{10} \times \frac{12}{17}$ **b** $\frac{3}{8} \times \frac{15}{26} \times \frac{8}{9}$ **c** $\frac{28}{5} \times \frac{11}{6} \times \frac{24}{7}$ **d** $1\frac{1}{12} \times \frac{8}{3} \times \frac{15}{4}$

e $\frac{22}{13} \times 10\frac{2}{5} \times 2\frac{1}{11}$ **f** $4\frac{1}{2} \times 5\frac{2}{5} \times \frac{21}{8}$ **g** $\frac{12}{5} \times 7\frac{1}{2} \times 3\frac{1}{4}$ **h** $6\frac{2}{3} \times 5\frac{1}{4} \times 1\frac{2}{7}$

9 Calculate the following. Hint: Remember the rules for the order of operations.

a $\frac{5}{8} + \frac{1}{2} \times \frac{3}{4}$ **b** $\frac{2}{3} \times \frac{4}{5} + \frac{8}{15}$ **c** $\frac{13}{6} - \frac{5}{3} \times \frac{5}{4}$ **d** $\frac{3}{8} + \frac{9}{4} \times \frac{11}{6} - \frac{8}{3}$

10 Toby has a garage sale where he hopes to sell 450 used books he has collected. He manages to sell $\frac{7}{9}$ of the books. How many books is he left with?

11 Hilaire works at the local swimming pool as a lifeguard. Her shift begins at 3:45 pm and ends at 6:15 pm.

a Write the hours Hilaire works in one shift as:

i a mixed number **ii** an improper fraction.

b If Hilaire has the same shift five nights a week, calculate the total number of hours she works as a mixed number.

12 While correcting his students' homework, a teacher found three different answers to the question:

'Find $2\frac{4}{9} \times 3\frac{3}{8}$ in its simplest form'. The answers given were: $6\frac{12}{72}$, $8\frac{1}{4}$ and $6\frac{12}{17}$.

a Which answer is correct?

b Comment on the error that is likely to have been made in each incorrect case.

13 A store takes $\frac{2}{5}$ off the price of its televisions during the stocktake sales. If a television normally costs $6320, what will it cost during the stocktake sale?

14 Amie manages to complete $\frac{3}{8}$ of her $5\frac{1}{2}$ km run before stopping. How far did she run prior to stopping?

15 Multiplying a number by a unit fraction is the same as dividing the number by the denominator of the unit fraction.

For example, $\frac{1}{2} \times 10 = \frac{10}{2}$

$= 10 \div 2.$

a Write the following multiplications as divisions.

i $\frac{1}{3} \times 33$ **ii** $\frac{1}{8} \times 40$ **iii** $\frac{1}{101} \times 55$ **iv** $\frac{1}{7} \times \frac{4}{9}$

b Write the following divisions as multiplications.

i $56 \div 7$ **ii** $90 \div 10$ **iii** $92 \div 35$ **iv** $\frac{8}{11} \div 5$

For any fraction, we can use the associative property to do the multiplication in a different order.

For example, $\frac{3}{2} \times 10 = 3 \times \left(\frac{1}{2} \times 10\right)$

$= 3 \times \frac{10}{2}$

$= 3 \times (10 \div 2)$

$= 3 \times 5$

$= 15.$

c Calculate the following using this method.

i $\frac{3}{8} \times 16$ **ii** $\frac{8}{5} \times 35$ **iii** $\frac{12}{7} \times 63$ **iv** $\frac{7}{6} \times \frac{18}{5}$

16 a Calculate each of the following products. Determine if the product is less than both fractions, greater than both fractions or between both fractions.

i $\frac{2}{3} \times \frac{4}{5}$

ii $3 \times \frac{1}{7}$

iii $\frac{9}{4} \times \frac{11}{3}$

iv $\frac{5}{6} \times 2\frac{5}{6}$

v $\frac{8}{3} \times \frac{3}{4}$

vi $\frac{1}{9} \times \frac{1}{7}$

vii $3\frac{1}{3} \times \frac{10}{3}$

viii $\frac{5}{3} \times 4$

b When will the product of two fractions be a value less than both fractions?

c When will the product of two fractions be a value greater than both fractions?

d When will the product of two fractions be a value between both fractions?

e What value does one of the fractions need to be so that the product is the same value as one of the fractions?

CHALLENGE

17 Saina's dad showed her the following multiplication:

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}$

Dad said, 'Wow, that took me ages to work out!'

Saina said, 'No Dad, this is really easy!'

How did Saina turn this into an easy calculation?

18 Consider the following multiplication pattern:

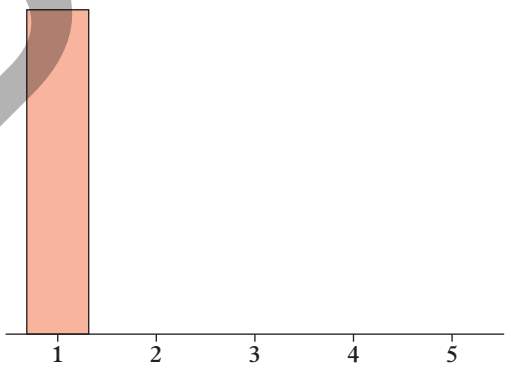
$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times \ldots$

a Determine the value of the number by multiplying up to the number $\frac{1}{5}$.

b Simplify the products in the table below.

Column	1	2	3	4	5
Value	$\frac{1}{1}$	$\frac{1}{1} \times \frac{1}{2}$	$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3}$	$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$	$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}$

c Complete the column graph below with the value of each column calculated in part **b**. For example, the value of column 1 is 1. Use that as one whole to determine the size of the other columns.



d As more fractions are multiplied in the pattern, does the result become greater or smaller?

e Estimate the whole number that the pattern will 'almost' produce but never actually give as a result.

Check your Student obook pro for these digital resources and more: pro

Worksheet Multiplying fractions

Investigation All things equal

Topic quiz 3F

3G Dividing fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify the reciprocal of a fraction
- ✔ divide proper and improper fractions
- ✔ divide mixed numbers.

Inter-year links

Year 8 — 2C Multiplying and dividing fractions

The reciprocal of a fraction

- When a fraction is multiplied by its **reciprocal**, the result is 1.

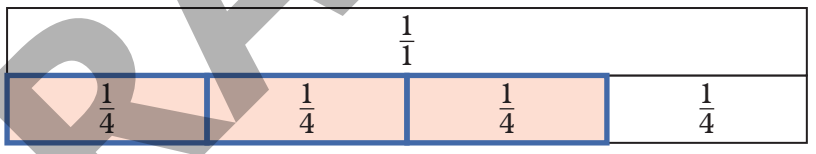
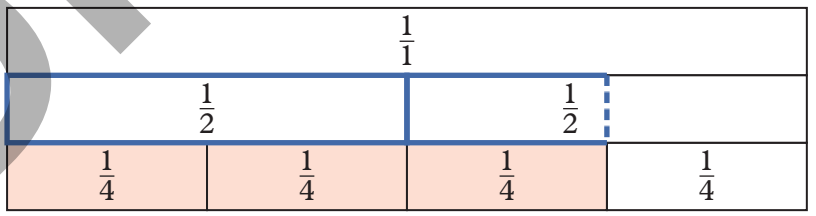
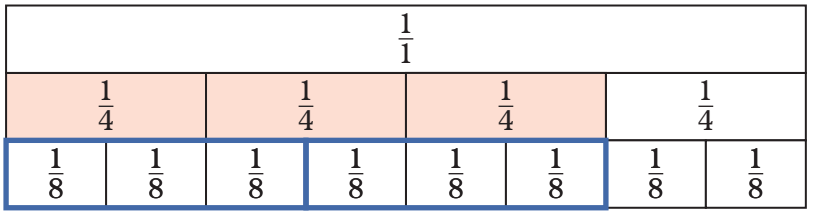
  For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ as $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.
- To find the reciprocal of a fraction, invert the fraction by swapping the numerator and the denominator.

Reciprocal

$\frac{3}{4} \rightleftarrows \frac{4}{3}$

Dividing fractions by fractions

- Dividing fractions can be thought of as asking: 'How many of one fraction fits into another fraction?'

$\frac{3}{4} \div \frac{1}{4} = 3$	How many $\frac{1}{4}$ are there in $\frac{3}{4}$?	
$\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$	How many $\frac{1}{2}$ are there in $\frac{3}{4}$?	
$\frac{3}{4} \div \frac{3}{8} = 2$	How many $\frac{3}{8}$ are there in $\frac{3}{4}$?	

- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction. Keep the first fraction the same, and then multiply by the reciprocal of the second fraction.

  For example, $\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3}$.

Dividing fractions by whole numbers

- Dividing by a whole number is the same as multiplying by the reciprocal of the whole number.
- A whole number can be written as a fraction with denominator 1, so the reciprocals of whole numbers are the unit fractions.

Reciprocal

$5 = \frac{5}{1}$ $\frac{1}{5}$

Dividing fractions by mixed numbers

- When dividing fractions by mixed numbers, start by converting the mixed numbers into improper fractions, then divide the fractions by multiplying by the reciprocal. Provide the result as a mixed number or proper fraction.

For example, $1\frac{2}{3} \div 2\frac{1}{4} = \frac{5}{3} \div \frac{9}{4}$

$= \frac{5}{3} \times \frac{4}{9}$

$= \frac{20}{27}$.

Example 3G.1 Finding the reciprocal of a fraction

Write the reciprocal of each value.

a $\frac{3}{7}$ **b** $\frac{1}{8}$ **c** 2 **d** $2\frac{3}{4}$

THINK

a Invert the fraction. Swap the numerator and the denominator.

b Invert the fraction. Simplify the result.

c Write the whole number as a fraction with a denominator of 1. Invert the fraction.

d Write the mixed number as an improper fraction. Invert the improper fraction.

WRITE

a $\frac{7}{3}$

b $\frac{8}{1} = 8$

c $2 = \frac{2}{1}$. The reciprocal is $\frac{1}{2}$.

d $2\frac{3}{4} = \frac{11}{4}$. The reciprocal is $\frac{4}{11}$.

Example 3G.2 Dividing fractions by fractions

Calculate $\frac{1}{7} \div \frac{3}{5}$.

THINK

1 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 Multiply the numerators together and then multiply the denominators together.

WRITE

$\frac{1}{7} \div \frac{3}{5} = \frac{1}{7} \times \frac{5}{3}$

$= \frac{1 \times 5}{7 \times 3}$

$= \frac{5}{21}$

Example 3G.3 Quotients between fractions and whole numbers

Calculate the following quotients:

a $\frac{4}{5} \div 6$ **b** $6 \div \frac{3}{8}$.

THINK

a **1** Keep the first fraction the same and multiply by the reciprocal of the second number. $6 = \frac{6}{1}$ so the reciprocal is $\frac{1}{6}$.

2 Cancel the 4 and 6 by dividing both numbers by the common factor 2.

3 Multiply the numerators together and then multiply the denominators together.

b **1** Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 3 is a factor of 6, so cancel the 3 in the denominator.

3 Multiple the numerator by 2.

WRITE

a $\frac{4}{5} \div 6 = \frac{4}{5} \times \frac{1}{6}$

$= \frac{\cancel{4}^{2}}{5} \times \frac{1}{\cancel{6}_{3}}$

$= \frac{2 \times 1}{5 \times 3}$

$= \frac{2}{15}$

b $6 \div \frac{3}{8} = 6 \times \frac{8}{3}$

$= \cancel{6}^{2} \times \frac{8}{\cancel{3}}$

$= 2 \times 8$

$= 16$

Example 3G.4 Dividing mixed numbers

Calculate $4\frac{3}{8} \div 2\frac{6}{7}$.

THINK

1 Convert each mixed number to an improper fraction.

2 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

3 Cancel the 35 and 20 by dividing each by the common factor 5.

4 Multiply the numerators together and then multiply the denominators together.

5 Change the improper fraction to a mixed number.

WRITE

$4\frac{3}{8} \div 2\frac{6}{7} = \frac{35}{8} \div \frac{20}{7}$

$= \frac{35}{8} \times \frac{7}{20}$

$= \frac{\cancel{35}^{7}}{8} \times \frac{7}{\cancel{20}_{4}}$

$= \frac{7 \times 7}{8 \times 4}$

$= \frac{49}{32}$

$= 1\frac{17}{32}$

Helpful hints

- ✔ The phrase 'Keep Change Flip' can help you remember how to turn a division problem into a multiplication problem.
- ✔ You can change any whole number into a fraction with a denominator of 1.
- ✔ Always simplify your fractions!

ANS p520

Exercise 3G Dividing fractions

1–11 | 4–7, 9–14 | 5–7, 9, 11–16

UNDERSTANDING AND FLUENCY

1 Rewrite each division problem in the form 'How many ____ are there in ____?'

a $2 \div \frac{1}{8}$ **b** $4 \div \frac{1}{3}$ **c** $\frac{1}{2} \div \frac{1}{12}$ **d** $\frac{1}{3} \div \frac{1}{9}$

2 Write the following statements as a division. Then, use the fraction wall to find the quotient.

a How many $\frac{1}{9}$ are there in $\frac{2}{3}$?

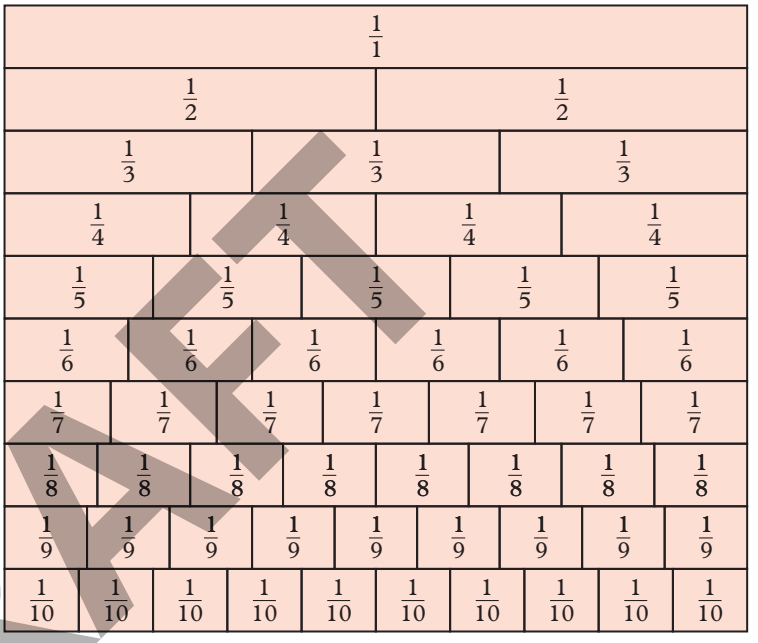
b How many $\frac{1}{5}$ are there in $\frac{3}{10}$?

c How many $\frac{2}{5}$ are there in $\frac{9}{10}$?

d How many $\frac{1}{3}$ are there in $\frac{5}{6}$?

e How many $\frac{3}{8}$ are there in 1?

f How many $\frac{2}{9}$ are there in $\frac{4}{6}$?



3G.1 **3** Write the reciprocal of each of these.

a $\frac{65}{31}$ **b** $\frac{11}{3}$

c $\frac{2}{5}$ **d** $\frac{27}{4}$

e $\frac{1}{100}$ **f** 8

g $\frac{34}{1}$ **h** $\frac{65}{3}$

i $4\frac{2}{7}$ **j** $9\frac{1}{8}$

4 Using the fact that dividing by a number is equivalent to multiplying by its reciprocal, write each of these divisions as multiplications. Do not perform the multiplication.

a $\frac{4}{7} \div \frac{5}{9}$ **b** $\frac{8}{3} \div \frac{1}{9}$ **c** $\frac{3}{5} \div 4$ **d** $6 \div \frac{3}{10}$ **e** $9\frac{3}{8} \div \frac{3}{2}$

f $\frac{8}{3} \div 6\frac{7}{10}$ **g** $5\frac{1}{5} \div 7\frac{4}{9}$ **h** $3 \div 2\frac{3}{8}$ **i** $4\frac{5}{8} \div 10$ **j** $21 \div 32$

3G.2 **5** Calculate each of the quotients. Give your answer as a mixed number if possible.

a $\frac{3}{8} \div \frac{2}{5}$ **b** $\frac{1}{12} \div \frac{1}{11}$ **c** $\frac{3}{4} \div \frac{3}{10}$ **d** $\frac{1}{9} \div \frac{4}{17}$ **e** $\frac{1}{10} \div \frac{4}{25}$

f $\frac{6}{7} \div \frac{8}{21}$ **g** $\frac{16}{15} \div \frac{12}{5}$ **h** $\frac{14}{3} \div \frac{7}{11}$ **i** $\frac{4}{13} \div \frac{20}{3}$ **j** $\frac{15}{27} \div \frac{11}{21}$

3G.3 **6** Calculate each of the quotients.

a $\frac{2}{9} \div 6$ **b** $\frac{3}{4} \div 9$ **c** $\frac{5}{2} \div 10$ **d** $\frac{8}{9} \div 2$ **e** $4 \div \frac{7}{5}$

f $6 \div \frac{1}{7}$ **g** $14 \div \frac{7}{6}$ **h** $12 \div \frac{8}{9}$ **i** $5 \div \frac{10}{13}$ **j** $18 \div \frac{21}{5}$

3G.4 **7** Calculate each of the quotients.

a $3\frac{1}{2} \div \frac{4}{5}$ **b** $1\frac{4}{7} \div \frac{5}{2}$ **c** $6\frac{1}{3} \div 1\frac{1}{4}$ **d** $7\frac{1}{5} \div \frac{22}{3}$

e $1\frac{2}{7} \div \frac{15}{21}$ **f** $2\frac{7}{9} \div \frac{4}{18}$ **g** $\frac{13}{4} \div 2\frac{6}{10}$ **h** $3\frac{3}{5} \div 1\frac{4}{5}$

i $4 \div 1\frac{1}{5}$ **j** $2\frac{1}{3} \div 5$ **k** $9\frac{1}{11} \div 10$ **l** $7\frac{3}{5} \div 19$

8 A batch of a chocolate pudding recipe requires $\frac{1}{6}$ kg of sugar. How many batches of chocolate pudding can be made from a 1 kg packet of sugar?

9 Over the school holidays, Brianna worked a total of 44 hours stacking shelves at the supermarket. If she only worked $5\frac{1}{2}$ hour shifts, how many shifts did she work?

10 Find the reciprocal of each answer from question **3**. Compare these to the original numbers. What can you conclude about finding the reciprocal of a number?

11 Rhys shares $3\frac{3}{8}$ pizzas equally between eight friends and himself. What fraction of a pizza will each person receive?

12 A new housing estate is to be developed on $43\frac{1}{5}$ hectares of land. If each block measures $\frac{3}{5}$ hectares, how many blocks will there be?

13 Gianluca must pack 15 kg of apples into plastic bags that can hold a maximum of $1\frac{1}{3}$ kg.

a How many $1\frac{1}{3}$ kg bags can he fill?

b How many kilograms of apples will be left over?

14 Priya and Rosa run a cafe that serves high tea on Sunday afternoons. Their menu states that each customer receives four finger sandwiches. Usually Priya and Rosa cut whole sandwiches into three fingers and the customers receive four fingers.

a Write the fraction of a whole sandwich each customer receives as an improper fraction.

b This Sunday they make 36 whole sandwiches. How many customers can they serve?

c They quickly realise they have more customers than expected, but they don't have enough ingredients to make any more sandwiches so decide to cut each sandwich into five fingers instead of three. How many customers could they serve with four fingers now?

d What fraction of a whole sandwich does each customer receive in this case?

15 Sava has started an online business selling backpacks that he makes from recycled T-shirts. He needs $1\frac{1}{3}$ T-shirts to make 1 backpack.

a How many T-shirts does he need to make 30 backpacks?

b At one of his favourite op shops Sava finds 15 T-shirts that are perfect for his backpacks. How many backpacks will he be able to make from these T-shirts?

16 Consider the number line below. If all the intervals are the same size, state whether the following are true or false.

a $A = 14$ **b** $B > 15$ **c** $C = 15\frac{3}{4}$ **d** $D < 17\frac{1}{2}$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Multiplying and dividing fractions

Investigation Paper cuts

Topic quiz 3G

PROBLEM SOLVING AND REASONING

CHALLENGE

3H Ratios

Learning intentions

By the end of this topic you will be able to ...

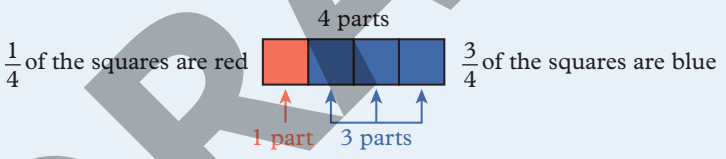
- ✔ compare quantities using ratios
- ✔ find and simplify equivalent ratios
- ✔ divide quantities into a ratio.

Inter-year links
Year 8 3E Ratios

Ratios

- A **ratio** is a comparison of two or more quantities of the same kind.
  - → Ratios are shown by a colon (:) meaning 'compared with' and showing the relationship between the equal parts.
    For example, the ratio 1 : 3 reads '1 part compared with 3 parts' or '1 to 3.'

- A ratio must be written in the order of the given worded description.
  For example, one part cordial to five parts water is written as 1 : 5.
- Ratios can be viewed as fractions where the numerator is the number in the ratio and the denominator is the total number of parts.

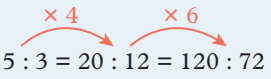


- Before writing a ratio, the numbers must be in the same unit of measurement.

Simplifying ratios

- A ratio is in its simplest form when the only common factor in the ratio is 1. Ratios can be simplified by:
  - → dividing the numbers in the ratio by the highest common factor,
  - → or by repeated division of common factors.

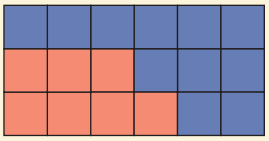
- Numbers in a ratio can be multiplied or divided by a value to create an equivalent ratio.



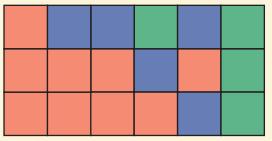
Example 3H.1 Determining ratios from a diagram

Write a ratio comparing the red to blue tiles in part **a** and red to blue to green tiles in part **b**.

a



b



THINK

1. Count the number of each coloured tile.
2. Write the numbers as they appear in the question: red then blue then green.
3. Write the ratio with a colon between each comparison.

WRITE

a There are 7 red tiles and 11 blue tiles.

red : blue

7 : 11

b There are 9 red tiles, 5 blue tiles and 4 green tiles.

red : blue : green

9 : 5 : 4

Example 3H.2 Simplifying ratios

Write the ratio 35 : 56 in its simplest form.

THINK

1. Find the highest common factor (HCF) of each number in the ratio.
2. Simplify by dividing each number in the ratio by the HCF.
3. Write the ratio.

WRITE

The HCF of 35 and 56 is 7.

35 : 56

÷ 7 ÷ 7

= 5 : 8

35 : 56 = 5 : 8

Example 3H.3 Dividing a quantity into a ratio

Divide 20 into the ratio 2 : 3.

THINK

1. Find the total number of parts in the ratio.
2. Divide the quantity by the total number of parts.
3. Multiply 4 by each part of the ratio.

WRITE

Number of parts = 2 + 3

= 5

20 ÷ 5 = 4

4 × 2 = 8

4 × 3 = 12

The ratio 2 : 3 divides 20 into 8 and 12.

Helpful hints

- ✔ Keep the order of the numbers in the ratio the same as the order of the worded comparison. It tells you the order of the quantities you are comparing.
- ✔ Ratios are not written in size order, only in comparison order.

ANS p521

Exercise 3H Ratios

△ 1–8, 9(a–d), 10–11, 14, 16, 18 □ 3–7, 8(d–f), 9, 11, 13, 15, 17, 18, 20–22 ⬠ 6, 7, 8(d–f), 9, 11, 13, 15, 19–25

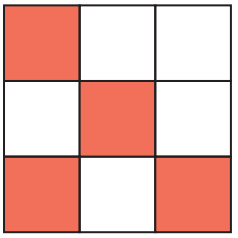
UNDERSTANDING AND FLUENCY

3H.1 **1** Write each comparison as a ratio of:

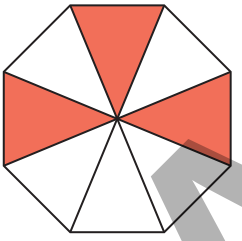
i the number of shaded parts to the number of non-shaded parts

ii the number of shaded parts to the total number of parts.

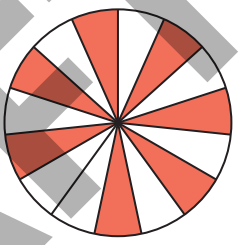
a



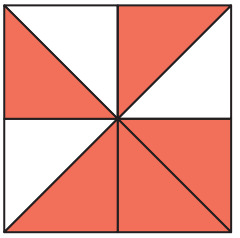
b



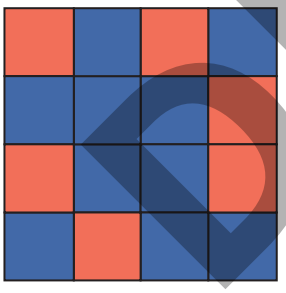
c



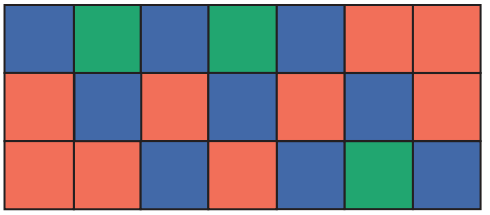
d



2 **a** What is the ratio of red to blue tiles?



b What is the ratio of blue to red to green tiles?



3 Write each comparison as a ratio:

a the number of red jelly beans compared with the number of yellow jelly beans

b the number of orange jelly beans compared with the total number of jelly beans

c the number of pink jelly beans compared with the number of yellow jelly beans compared with the number of blue jelly beans

d the number of red and blue jelly beans compared with the number of pink and purple jelly beans.

4 Write each comparison as a ratio in the given order.

a 132 cm to 207 cm **b** 16 kg to 35 kg

c 18 min to 11 min **d** $85 to $121

5 Write each comparison as a ratio in the given order.

a Kayla spent 5 hours on Facebook and 1 hour doing homework.

b Australia won three gold, four silver and nine bronze medals in the World Swimming Championships.

c Twenty-eight students tried out for the volleyball team, while 23 students tried out for the netball team.

d To make a drink of cordial, use one part cordial to four parts water.

3H.2 **6** Write each ratio in its simplest form.

a 21 : 6	**b** 12 : 18	**c** 52 : 64	**d** 36 : 96
e 88 : 48	**f** 40 : 45	**g** 65 : 13	**h** 22 : 132

7 A fruit bowl contains apples, mandarins and bananas and is shown in the image to the right. There are three bananas, three apples and one mandarin.

Write each comparison as a ratio in its simplest form:

a the number of apples to the number of mandarins

b the number of apples to the number of bananas

c the number of mandarins to the number of bananas to the number of apples.

8 Complete the patterns of equivalent ratios.

a 3 : 2 6 : 4 15 : __	**b** 4 : 5 __ : 25 16 : __	**c** __ : 210 10 : 70 2 : __
d 144 : __ 24 : 9 8 : __	**e** __ : 121 150 : __ 15 : 11	**f** 147 : __ __ : 63 7 : 9

3H.3 **9** Divide the following amounts into the given ratio.

a 33 in 5 : 6	**b** 35 in 3 : 2	**c** 70 in 2 : 5	**d** 132 in 2 : 1
e 119 in 9 : 8	**f** 195 in 2 : 13	**g** 2304 in 8 : 1	**h** 1554 in 5 : 9

10 Max scored $\frac{87}{100}$ (or 87 out of 100) for his Geography test.

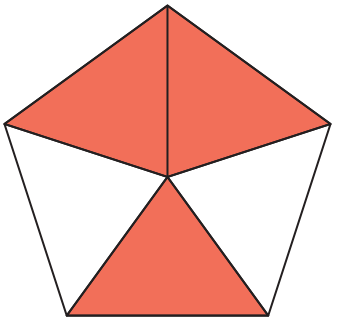
Write a ratio for:

a the marks received compared to the total number of marks

b the marks lost compared to the total number of marks

c the marks received compared to the marks lost.

11 Look at the pentagon shown.



a What is the ratio of shaded sections to non-shaded sections?

b What is the ratio of non-shaded sections to the total number of sections?

c What is the ratio of shaded sections to the total number of sections?

d Can any of the ratios in parts **a–c** be written as fractions? Explain your reasoning.

e Can any of the ratios in parts **a–c** *not* be written as fractions? Explain your reasoning.

f True or false: Ratios can be written as fractions when comparing the ratio of a part to its whole.

12 In a bag of 28 red and blue marbles, 16 marbles are red.

a How many marbles are blue?

b Write the ratio of red to blue marbles in its simplest form.

13 Jesse and James bought a $13 lottery ticket together. James was a bit short on cash, so Jesse paid $8 and James paid the rest. Against all odds, they won $26 million dollars. They decide to share the money in the same ratio that they paid for the ticket. How much money do Jesse and James each get?

14 A breakfast cereal mix is made using 8 kg of oats, 4 kg of dried fruit and 12 kg of wheat flakes.

a Write the ratio of oats to dried fruit to wheat flakes in its simplest form.

b Write each ingredient as a fraction of the whole mix.

15 Geoff mixes 4 L of blue paint with 5 L of yellow paint to create a shade of green for his studio.

a Write the ratio of the amount of blue paint to the amount of green paint.

b How much green paint does Geoff have?

c What fraction of the new shade is the blue paint?

d A year later, Geoff notices marks on the wall that won't wipe off. He decides to repaint that area. If he bought a 2 L tin of blue paint and a 6 L tin of yellow paint, how much of the same shade of green paint can Geoff make?

16 Last week, Shannen completed extra chores around home and earned $32. This week, she earned $14 less. Write as a ratio, in its simplest form, the amount of last week's pocket money to this week's pocket money.

17 The table shows the attendance at each performance of the local school production.

Performance	Number of people attending
Thursday evening	486
Friday evening	549
Saturday matinee	390
Saturday evening	520

a Find the total attendance at the school production over the three days.

b Write as a ratio in its simplest form:

i the number attending Thursday's performance to the number attending Friday's performance

ii the number attending Saturday's matinee performance to the number attending Saturday's evening performance.

c If the venue can seat a maximum of 550 people, write as a ratio in its simplest form the total attendance to the total possible attendance over the three days.

18 Is each pair of ratios equivalent? Give a reason for your answer.

a 7 : 8 and 21 : 24

b 20 : 32 and 5 : 8

c 63 : 18 and 9 : 2

d 30 : 45 and 90 : 60

19 A two-ingredient dough recipe uses $1\frac{1}{4}$ cups of self-raising flour to 1 cup of Greek yoghurt and makes four servings. How many cups of self-raising flour and Greek yoghurt are needed to make 10 servings? Write your answers using mixed numbers.

20 The ratio of apple juice to soda water for a fruit punch recipe is 5 : 3. If the recipe requires 750 mL of apple juice, how much soda water is required?

21 Divide $300 using the ratio 4 : 6.

a Calculate the total number of parts in the ratio.

b Write each number in the ratio as a fraction of the total number of parts.

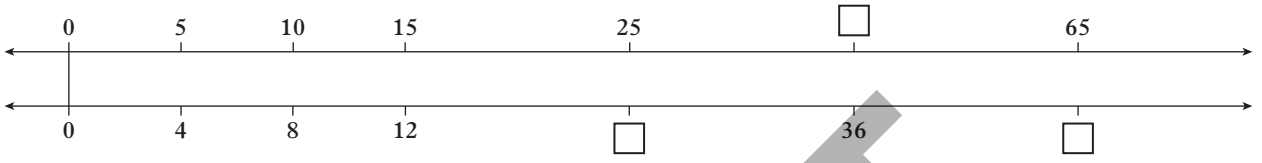
c Multiply each fraction from part **b** by the quantity ($300).

d Do the amounts found in part **c** add to $300?

e Decide if you prefer this method of dividing a quantity into a ratio.

22 Nadia divides her pocket money for April in the ratio 3 : 2 : 1 to cover buying a gift, entertainment expenses and savings. Calculate the amount she has allocated for entertainment expenses if her total pocket money for April is $84.

23 Below is a double number line that is being used to show the relationship between the ratio 5 : 4 and ratios that are equivalent to it.



This can also be shown in a table. See below.

0	5	10	15	25		65
0	4	8	12		36	

a Complete the double number line and ratio table.

b There are two calculations that produce the equivalent ratios 5 : 4, 10 : 8 and 15 : 12.

We can see that $5 \times 3 = 15$ and $4 \times 3 = 12$, but also $5 + 10 = 15$ and $4 + 8 = 12$.

Explain why both pairs of calculations move between the equivalent ratios 5 : 4 and 15 : 12.

24 A bag has a number of green and orange marbles in it. The ratio of green to orange marbles is 5 : 8. When four green marbles are taken out of the bag, the ratio changes to 3 : 5. Find the number of marbles originally in the bag.

25 Itsuki has two tins of paints in different shades of purple. The first is a mix of purple and white paint in the ratio 1 : 1 in a 1 L tin. The second is a mix of purple and white paint in the ratio 3 : 1 in a 4 L tin.

a Which tin of paint has a darker shade?

b If Itsuki mixes both tins of paint together, how many litres of paint will she have?

c How much white paint is in the mixture?

d What is the ratio of purple and white paint in the mixture?

Itsuki's friend Ito has two tins of paint as well. The first is a mix of purple and white paint in the ratio 4 : 1 in a 1 L tin. The second is a mix of purple and white paint in the ratio 1 : 1 in a 4 L tin.

e If Ito wants to add paint from the second tin to the first tin to match the colour of Itsuki's paint, what fraction of the second tin's paint should he add?

f How much paint is left in the second tin?

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Visual representations of ratios

Interactive skillsheet Simplifying ratios

Interactive skillsheet Dividing a quantity into a ratio

Investigation Quilt patterns

Topic quiz 3H

Chapter summary

Types of fractions

- Proper fractions
  $\frac{3}{5}, \frac{2}{6}, \frac{11}{15}, \frac{3}{3}$
- Improper fractions
  $\frac{13}{6}, \frac{9}{5}, \frac{31}{8}$
- Mixed numbers
  $2\frac{1}{6}, 1\frac{2}{3}, 4\frac{5}{6}$

Equivalent fractions

$\frac{2}{3} = \frac{10}{15}$ ($\times 5$ above, $\times 5$ below) $\qquad \frac{12}{10} = \frac{6}{5}$ ($\div 2$ above, $\div 2$ below)

- The HCF of 36 and 72 is 36,

$$\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}} = \frac{1}{2}$$

Improper fractions to mixed numbers

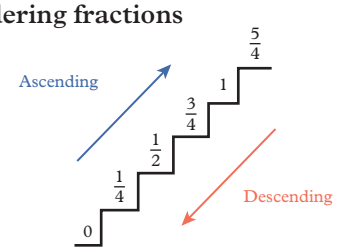
$\frac{13}{4} = 13 \div 4 = 3$ remainder 1

$= 3\frac{1}{4}$

Mixed numbers to improper fractions

$$3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$$

Ordering fractions



- To compare $\frac{1}{2}$ and $\frac{3}{5}$, find the LCD and multiply the numerator and denominator of each fraction to get an equivalent fraction. Then compare the fractions with the same denominator.

Add and subtract like fractions

1. Add or subtract the numerators and keep the same the denominator.
2. Simplify the fraction.

$$\frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

Add and subtract unlike fractions

1. Convert into equivalent fractions with a common denominator.
2. Add or subtract the numerators and keep the same the denominator.
3. Simplify the fraction.

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$$

Add and subtract mixed numbers

1. Convert mixed numbers into improper fractions.
2. Convert the improper fractions into equivalent fractions with a common denominator.
3. Add or subtract the numerators and keep the same denominator.
4. Convert the answer into a mixed number.
5. Simplify the fraction.

$$2\frac{1}{4} + 3\frac{1}{5} = \frac{9}{4} + \frac{16}{5} = \frac{45}{20} + \frac{64}{20} = \frac{109}{20} = 5\frac{9}{20}$$

Multiplying fractions

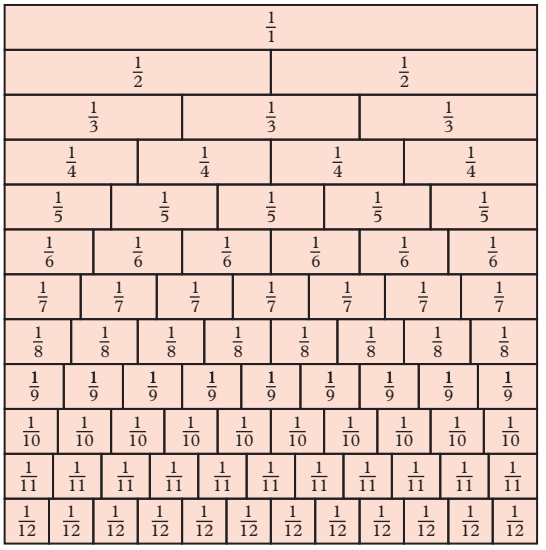
1. Multiply the numerators together.
2. Multiply the denominators together.
3. Simplify the fraction.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

Reciprocal

$\frac{3}{4}$ and $\frac{4}{3}$ $\qquad 5 = \frac{5}{1}$ and $\frac{1}{5}$

Fraction wall



Dividing fractions

- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.
- Keep the first fraction the same, and multiply by the reciprocal of the second fraction.

$$\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3}$$

Simplifying ratios

120 : 72

$\div 24$ $\qquad$ $\div 24$

5 : 3

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- cancelling
- common denominator
- denominator
- equivalent fraction
- fraction
- fraction wall
- highest common factor (HCF)
- improper fraction
- like fractions
- lowest common denominator (LCD)
- mixed numbers
- number line
- numerator
- proper fraction
- ratio
- reciprocal
- set
- simplest form
- simplify
- unit fraction
- unlike fractions

1 What are the differences between like fractions and unlike fractions?

2 Use fractions to represent the following quantities.

a a half **b** two-thirds

c three quarters **d** four and one-tenth

3 Write the following fractions in words.

a $\frac{1}{3}$ **b** $\frac{2}{9}$ **c** $\frac{2}{4}$ **d** $12\frac{1}{2}$

4 Identify the key terms being referenced in each of these definitions.

a fractions that have the same numerical value

b a common multiple of the denominators of two or more fractions

5 Provide a definition in your own words for the following key terms using an example.

a unit fraction **b** reciprocal **c** ratio

6 Complete the following sentences using words from the key term list.

a _____ have a numerator that is greater than or equal to the denominator. They can be written as _____, which consist of both a whole number and a _____.

b When a fraction is written in _____, the_____ between the numerator and the denominator is 1.

c When adding and subtracting like fractions, add and subtract the_____ and keep the same _____.

7 Identify whether the following statements are true or false. If a statement is false, briefly explain why.

a A proper fraction has infinitely many equivalent fractions.

b All whole numbers can be written as fractions.

c Number lines and fraction walls can both be used to compare fractions.

d A unit fraction cannot be further simplified.

e The reciprocal of 0 is 0.

f Cancelling can only be performed when a fraction is multiplied by a whole number.

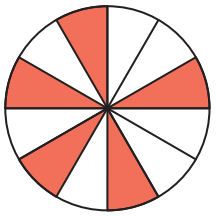
g A ratio can be used to compare up to three quantities.

Multiple choice

3A **1** A worksheet has 29 questions on it. If you have completed 17 of the questions, what fraction of the worksheet is complete?

A 17 **B** $\frac{29}{17}$ **C** 29 **D** $\frac{1}{2}$ **E** $\frac{17}{29}$

3A **2** What fraction of the shape is coloured red?



A 5 **B** $\frac{5}{10}$ **C** $\frac{7}{12}$ **D** $\frac{5}{12}$ **E** 7

3B **3** Which of the following is an equivalent fraction to $\frac{2}{5}$?

A $\frac{3}{5}$ **B** $\frac{6}{15}$ **C** $\frac{2}{10}$ **D** $\frac{1}{10}$ **E** $\frac{5}{15}$

3B **4** Simplify $\frac{120}{195}$.

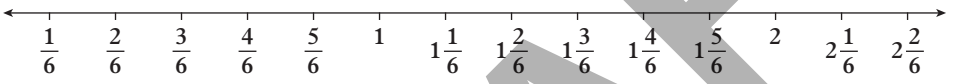
A 15 **B** $\frac{8}{13}$ **C** $\frac{40}{65}$ **D** $\frac{24}{39}$ **E** $\frac{1}{1.625}$

3C **5** $1\frac{2}{6}, \frac{2}{3}, 1\frac{1}{2}, \frac{5}{12}$ in descending order is

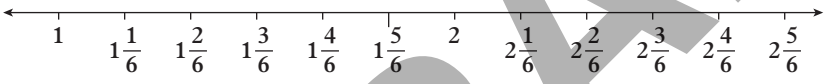
A $1\frac{1}{2}, 1\frac{2}{6}, \frac{2}{3}, \frac{5}{12}$ **B** $\frac{5}{12}, \frac{2}{3}, 1\frac{2}{6}, 1\frac{1}{2}$ **C** $1\frac{2}{6}, 1\frac{1}{2}, \frac{5}{12}, \frac{2}{3}$ **D** $\frac{2}{3}, \frac{5}{12}, 1\frac{1}{2}, 1\frac{2}{6}$ **E** $\frac{5}{12}, 1\frac{2}{6}, \frac{2}{3}, 1\frac{1}{2}$

3D **6** Which of the following number lines is the most appropriate to compare the following fractions: $\frac{5}{6}, \frac{1}{3}, 1\frac{1}{3}, \frac{7}{6}, \frac{7}{3}$?

A

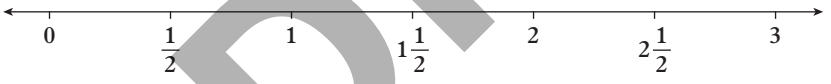


B

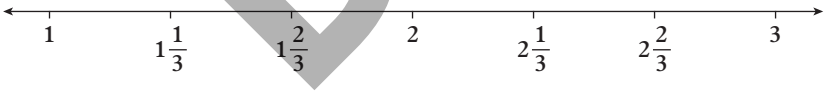


C

D



E



3D **7** Convert $3\frac{4}{7}$ to an improper fraction.

A $\frac{7}{7}$ **B** $\frac{4}{10}$ **C** $\frac{16}{7}$ **D** $\frac{25}{7}$ **E** $2\frac{11}{7}$

3E **8** What is $\frac{2}{5} + \frac{1}{3}$ equal to?

A $\frac{3}{8}$ **B** $\frac{2}{15}$ **C** $\frac{1}{2}$ **D** $\frac{11}{15}$ **E** $\frac{1}{15}$

3E **9** What is $1\frac{1}{2} + 2\frac{5}{6} - 1\frac{2}{3}$?

A $2\frac{2}{3}$ **B** 4 **C** $2\frac{4}{5}$ **D** 6 **E** $\frac{3}{8}$

3F **10** Which of the following pairs of fractions has a lowest common denominator that is not equal to the product of the denominators?

A $\frac{4}{12}, \frac{6}{7}$ **B** $\frac{3}{2}, \frac{2}{3}$ **C** $\frac{5}{8}, \frac{11}{12}$ **D** $\frac{14}{1}, \frac{2}{5}$ **E** $\frac{5}{18}, \frac{9}{35}$

3F **11** What is $\frac{2}{7}$ of $\frac{8}{9}$?

A $\frac{16}{63}$ **B** $\frac{10}{16}$ **C** $\frac{6}{2}$ **D** $\frac{18}{56}$ **E** $\frac{74}{63}$

3F **12** What is $\frac{14}{5} \times \frac{3}{21} \times \frac{25}{30}$ in its simplest form?

A 3 **B** $\frac{5}{3}$ **C** $\frac{3}{5}$ **D** $\frac{1}{3}$ **E** $\frac{1}{2}$

3G **13** What is $\frac{7}{20} \div \frac{14}{5}$ equal to?

A $\frac{1}{2}$ **B** $\frac{1}{8}$ **C** $\frac{7}{15}$ **D** $\frac{21}{25}$ **E** $\frac{49}{50}$

3H **14** What is a mixture of 100 mL cordial to 1.4 L of water written as a ratio in its simplest form?

A 100 : 1.4 **B** 100 : 1400 **C** 1 : 14 **D** 101.4 **E** 5 : 70

3H **15** Dario and Barbosa divide \$150 in the ratio 3 : 2. Dario will get:

A \$5 **B** \$60 **C** \$30 **D** \$90 **E** \$50.

Short answer

3A **1** Mark the following fractions on a number line between the given values.

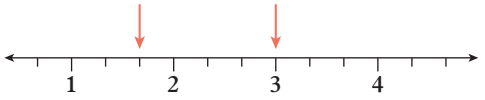
a $1\frac{3}{4}$ on a number line from 0 to 2

b $\frac{7}{12}$ on a number line from 0 to 1

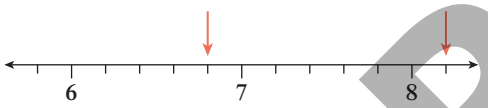
c $\frac{16}{3}$ on a number line from 2 to 6

3A **2** Write the improper fractions identified by the arrows.

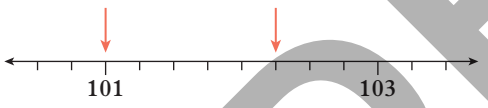
a



b



c



3B **3** For each of the following fractions, write an equivalent fraction using the given information.

a $\frac{2}{5}$ with a denominator of 25

b $\frac{9}{8}$ with a numerator of 27

c $\frac{3}{7}$ with a numerator of 21

d $\frac{50}{120}$ with a denominator of 24

3B **4** Simplify each fraction.

a $\frac{16}{20}$ **b** $\frac{5}{45}$ **c** $\frac{50}{100}$ **d** $\frac{21}{49}$

3C **5** Place these fractions in ascending order.

$\frac{1}{4}, \frac{2}{7}, \frac{3}{5}, 1\frac{2}{3}, \frac{9}{2}, \frac{11}{20}$

3D **6** Complete this table.

	Mixed number	Improper fraction
a	$2\frac{3}{8}$	
b		$\frac{11}{2}$
c		$\frac{17}{9}$
d	$5\frac{1}{3}$	

3E **7** Calculate:

a $\frac{1}{9}+\frac{4}{9}$ **b** $\frac{7}{11}-\frac{5}{11}$ **c** $\frac{2}{3}-\frac{1}{2}$ **d** $\frac{3}{10}+\frac{2}{5}$

3E **8** Calculate:

a $2\frac{5}{7}+3\frac{1}{7}$ **b** $2\frac{1}{5}-1\frac{2}{5}$ **c** $1\frac{3}{8}-\frac{1}{4}$ **d** $1\frac{4}{5}+2\frac{2}{3}$

3F **9** Calculate:

a $\frac{2}{5}\times\frac{4}{7}$ **b** $\frac{3}{10}\times\frac{5}{9}$ **c** $\frac{2}{3}\times\frac{10}{11}$ **d** $\frac{12}{5}\times\frac{4}{9}$

3F **10** Calculate:

a $\frac{2}{3}\times1\frac{4}{5}$ **b** $2\frac{3}{7}\times\frac{1}{5}$ **c** $2\frac{1}{4}\times3\frac{2}{3}$ **d** $1\frac{3}{8}\times4\frac{1}{3}$

3G **11** Calculate:

a $\frac{3}{4}\div\frac{2}{3}$ **b** $\frac{25}{12}\div\frac{5}{3}$ **c** $\frac{49}{24}\div\frac{7}{16}$ **d** $\frac{1}{4}\div\frac{1}{3}$

3G **12** Calculate:

a $3\frac{4}{7}\div2\frac{1}{7}$ **b** $\frac{8}{9}\div1\frac{1}{3}$ **c** $3\frac{3}{5}\div\frac{6}{15}$ **d** $2\frac{3}{5}\div1\frac{9}{15}$

3H **13** Use the photo of the Smarties to write each comparison as a ratio.

a the number of pink Smarties to the number of yellow Smarties

b the number of yellow Smarties to the number of green Smarties

c the number of orange Smarties to the total number of Smarties

d the number of orange and pink Smarties to the number of green Smarties

3H **14** Write as ratios in their simplest form.

a 24 : 36 **b** 15 : 20 **c** 75 : 120 **d** 56 : 4

3H **15** The ratio of snakes to lizards in a reptile zoo is 7 : 5. If there are 40 lizards, how many snakes are there in the zoo?

Analysis

1 A survey of 300 families found that 150 have one pet and 100 have two pets.

a What fraction of the families have one pet? Write this in its simplest form.

b What fraction of the families have two pets? Write this in its simplest form.

c Write a ratio for the number of families with one pet compared to those with two pets.

d If $\frac{1}{12}$ of the families have three or more pets, how many families is this?

e What fraction of the families do not have pets? How many families is this?

f A news reporter said that 5 in every 10 people have one pet, and 3 in every 10 people have two pets. Are they correct? Use your knowledge of equivalent fractions to support your answer.

2 Lili, Camila and Madelaine each bought half a cake in their favourite flavour.

a What fraction of a whole cake do they have altogether?

b Lili ate $\frac{1}{3}$ of her half-cake, Camila ate $\frac{3}{5}$ of her half-cake, and Madelaine ate $\frac{7}{15}$ of her half-cake. What fraction of a whole cake have they eaten in total?

c What fraction of a whole cake do they have left?

d What fraction of the amount of cake they started with do Lili, Camila and Madelaine have left?

e Their friend Casey says he can eat $\frac{3}{4}$ of the leftover cake. How much of a whole cake does Casey think he can eat?

f If Casey does eat $\frac{3}{4}$ of the leftover cake and Lili, Camila and Madelaine cannot ever eat more cake than they did in part **b**, who would be able to eat the rest of the cake tomorrow?

3 Keith found two bottles of 2 L juice in the fridge. One bottle is half empty, and the other is unopened.

a How much juice is left in the half empty bottle?

b Keith poured all the juice from the half empty bottle into a 3 L jug. What fraction of the jug is full?

c Keith drank $\frac{1}{4}$ L of juice and his friend Emma drank $\frac{1}{2}$ L of juice from the jug. How much juice is left in the jug?

d What fraction of the jug is empty?

e Emma opened the second bottle of juice to refill the jug to half full. How much juice did Emma add?

f What fraction of the juice bottle is full after the refill?

g Keith and Emma shared the juice in the jug in the ratio of 3 : 2. How much juice did Keith have in total?

Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify fractions from images, statements and number lines ☐ Express one quantity as a fraction of another	☐ Go back to Topic 3A Fractions
☐ Recognise equivalent fractions on a fraction wall and number line ☐ Identify and find equivalent fractions ☐ Simplify fractions	☐ Go back to Topic 3B Equivalent fractions
☐ Compare fractions with the same numerator or denominator ☐ Order fractions in ascending and descending order ☐ Order and compare fractions using equivalent fractions	☐ Go back to Topic 3C Ordering fractions
☐ Identify mixed numbers and improper fractions on a number line ☐ Convert improper fractions to mixed numbers ☐ Convert mixed numbers to improper fractions	☐ Go back to Topic 3D Mixed numbers and improper fractions
☐ Add and subtract fractions with the same denominator ☐ Add and subtract fractions with different denominators ☐ Add and subtract mixed numbers	☐ Go back to Topic 3E Adding and subtracting fractions
☐ Multiply fractions by whole numbers ☐ Multiply fractions by fractions ☐ Multiply mixed numbers	☐ Go back to Topic 3F Multiplying fractions
☐ Identify the reciprocal of a fraction ☐ Divide proper and improper fractions ☐ Divide mixed numbers	☐ Go back to Topic 3G Dividing fractions
☐ Compare quantities using ratios ☐ Find and simplify equivalent ratios ☐ Divide quantities into a ratio	☐ Go back to Topic 3H Ratios

Contact us

For more information about this series please contact a secondary education consultant in your state or territory:

Northern Territory

Melissa Wearne
E: melissa.wearne@oup.com
M: 0447 557 931

Western Australia

Graeme Jordan
E: graeme.jordan@oup.com
M: 0417 308 540

South Australia

Renee Kennedy
E: renee.kennedy@oup.com
M: 0418 857 546

Victoria

Alicia Giaquinta
E: alicia.giaquinta@oup.com
M: 0411 759 611

Caitlin Hansford
E: caitlin.hansford@oup.com
M: 0411 759 659

Paul McCallum
E: paul.mccallum@oup.com
M: 0423 241 455

Tasmania

Caitlin Hansford
E: caitlin.hansford@oup.com
M: 0411 759 659

Queensland

Chris Lowcock
E: christopher.lowcock@oup.com
M: 0428 949 935

Melissa Wearne
E: melissa.wearne@oup.com
M: 0447 557 931

New South Wales

Caly James
E: caly.james@oup.com
M: 0413 745 855

Sue Dowd
E: sue.dowd@oup.com
M: 0411 759 608

Australian Capital Territory

Caly James
E: caly.james@oup.com
M: 0413 745 855

Visit us at **oup.com.au**
or contact customer support at **oup.com.au/help**

oup.com.au
@OxfordAustraliaNZ
facebook.com/oupanz
linkedin.com/showcase/oxford-university-press-australia-new-zealand

OXFORD UNIVERSITY PRESS